UDC 534.21

ON THE ACOUSTIC RADIATION FORCE CALCULATION FOR RIGID SPHERICAL PARTICLE LOCATED IN THE FLUID-FILLED CYLINDRICAL TUBE WITH RIGID OR ELASTIC WALLS

O. P. Zhuk[†1], **V. D. Kubenko**[1], **Y. O. Zhuk**[‡2]

[1]*S. P. Timoshenko Institute of Mechanics of NAS of Ukraine*
3, Petra Nesterova Str., 03057, Kyiv, Ukraine
[†]*E-mail: zhuk@inmech.kiev.ua*

[2]*Taras Shevchenko National University of Kyiv*
64/13, Volodymyrs'ka Str., 01601, Kyiv, Ukraine
[‡]*E-mail: y.zhuk@i.ua*

Received 02.07.2024 ⋄ Accepted 20.03.2025 ⋄ Published 03.09.2026

The acoustic pressure and acoustic radiation force (ARF) acting on the spherical rigid particle immersed in a fluid filling the rigid or elastic cylindrical tube are calculated. An expression for ARF is deduced for the case when the incident wave is a plane wave propagating along the axis of the tube. Both acoustic pressure and ARF are determined as the function of the primary wave frequency. The solution of the problem stated is derived with making use of the previously derived expressions for a plane harmonic wave scattering on a spherical body. The problem is attacked with the application of the variable separation technique. The most complex part of the problem is the boundary conditions satisfaction on cylindrical and spherical surfaces. The mutual expansions of spherical wave functions over the cylindrical ones and vice versa are used for this purpose. An exact analytical solution was derived as the special function expansion. The coefficients of the expansion are calculated using the infinite system of the algebraic equations, which is solved by a truncation method. Simulation results demonstrate that the ARF value is significantly affected by the presence of the cylindrical boundary surface. It is caused by the influence of the acoustic wave scattered by the cylindrical surface of the tube. The sharp resonance-like peaks occur at the same positions in the ARF vs. incident wave frequency curves. This effect is caused by the resonance-like behavior of the fluid-filled cylinder, i. e., the peaks' presence at some discrete values of the frequency is stipulated by specific localization effects and influences the motion of the particles in the cylindrical fluid-filled cavity. It is also found that depending on the frequency of an incident wave, the ARF can change its direction. This effect provides the scientific basis for the particle manipulation technique if the ones are located in the fluid-filled tubes with both rigid and elastic walls.

KEY WORDS: acoustic radiation force, cylindrical tube, spherical particle, ideal compressible fluid, plane harmonic wave

DOI: ***

1. INTRODUCTION

The plane harmonic wave propagating in the acoustic medium induces the periodic component and constant component of the sound pressure. The latter one, being averaged over the target's surface and then over the incident wave period, is usually referred to as the acoustic radiation force (ARF) (see [1–3]). The physical nature of the ARF can be explained by the following reasoning [4]: the radiation pressure is exerted by an acoustic field provided that an average impulse transferred by the wave for the period of oscillation varies in some volume of medium resulting in the generation of the stationary component of the sound pressure acting over this volume. The ARF, being acoustically induced hydrodynamic force, is the cause of the series of interesting and technologically valuable phenomena observed for the particles, bubbles and other obstacles suspended in the fluid. These phenomena are an essential ingredient of huge amount of recent scientific, technological, and medical applications, such as ARF imaging, colloidal assembly, and various bio-separation assays [1, 2, 5]. The sound pressure is employed to create the so called 'acoustic levitation effect', and to elaborate the technique for contactless particle 'suspension' and even contactless manipulation or fixing of the 'swarm' of particles in the certain regions [6–8] which can be of a great importance for the medical applications handling the drug delivery problems particularly.

ARF describes the measure of interaction between a wave and a particle. Theoretical examination of the ARF effect on the obstacles is a complicated problem. It is necessary to account for a set of instances to attack it properly: the shape of the obstacle, its size with regard of the wave length, the properties of a fluid, and boundary conditions on the interface between the fluid and external medium etc. Simulation of the ARF influence on a obstacle was the subject of numerous studies over decades [3, 9–11]. Initially, the ARF induced by a plane wave acting on spheres or cylinders in an unbounded medium was studied for example in the works by [12–14]. It was King who has initially proposed the theory of acoustic radiation pressure acting on a sphere in a compressible fluid [3]. Fox used a simple approximation to obtain the expression of ARF function for a rigid sphere [15], Faran studied the scattering by solid cylinders and spheres [16], while Doinikov [14] extended the theory by considering a sphere in a viscous medium. In the work [17], an expression for the acoustic radiation pressure on solid elastic particles was deduced in an unbounded fluid. This paper contains the results of experimental investigations for elastic spheres as well.

The much more complicated problems of the ARF acting upon a spherical particle located in fluid-filled bounded regions were addressed in [4, 18]. The main objects investigated were spherical particle located in cylindrical cavity (tube) or half-space at the vicinity of a rigid wall mainly. Relatively few models have been developed for the acoustic radiation forces acting on nonspherical objects, particularly based on analytical methods because the models of ARF have been developed primarily for the interaction of a sound field with planar and spherical surfaces. The review of the traditional approaches can be found in [19] for instance.

Now, the approaches ranging from exact analytical solution and its approximations [3, 20–22] to numerical simulation [6, 8, 23] are applied to attack the ARF determination problem. As for the types of the incident waves investigated, the plane traveling wave, plane standing wave, dual orthogonal standing waves, acoustic beams with arbitrary wavefront and helicoidal Bessel beam were studied in the works [8, 10, 11, 20, 22–24] to name a few. The investigated acoustic media were ideal and viscous fluids. Although analytical approaches proved to be effective even in the cases of the wall proximity to the cylindrical and spherical particles [11,20,21], the considerable attention was also paid to the numerical techniques which are particularly useful for the complex obstacles shapes, complicated viscous functions, acoustic streaming etc [6–8, 23].

The influence of the fluid space limitation on the ARF is investigated for a case of rigid particles located in the vicinity of the rigid plane boundary of a fluid (see papers [11,20,21,25] or in the fluid filled tube with rigid walls [4, 18]. It worth mentioning, the accounting for compliance of the bounding surface (walls) is real challenge. When the properties of the fluid and of the tube are uniform along it, the linear theory leads to the same one-dimensional wave equation as for the plane sound waves, but with the value of the wave velocity affected by the distensibility of the tube [26]. In fact, effective compressibility of the fluid in the tube is a sum of the true fluid compressibility and distensibility of the tube. In the present paper, the problems of ARF determination for the rigid spherical particle placed in a cylindrical tube with rigid as well elastic walls filled by ideal compressible fluid is under consideration. The longitudinal plane wave propagates along the tube and scatters on the particle and cylindrical wall. It is also supposed that the particle is centered on the cylinder axis so the rotational symmetry is of the case. According to the approach developed earlier in the papers [4, 20], solution of the specified problems are sought for in two basic steps. The first step comprises the examination of regularities of interaction between incident harmonic sound wave and obstacles: a sphere and a tube surface, i. e,. the solution of the problem of a primary wave scattering. With making use of the results obtained, the second step encompasses determining of the hydrodynamic forces acting on the rigid sphere with their subsequent averaging over the suitable time interval to obtain the ARF value.

2. VELOCITY POTENTIAL DETERMINATION PROBLEM

ARF determination problems for the case of a target suspended in an ideal fluid are normally formulated with making use of the Lagrange coordinate system. Therefore, as it was mentioned above, the radiation pressure is defined as time-averaged value of the acoustic pressure over the target surface. This approach is proved to be applicable for the deviation of the pressure imposed by the sound wave if the harmonic law with respect to time is assumed in the obstacle vicinity [1–3]. Linear approximation for the radiation pressure is insufficient because having the sound pressure as periodic function of time being averaged over the wave period gives zero value of the force [27]. Therefore, correct estimation of the ARF demands the quadratic terms in a wave equation should be preserved as well. Hence, second order approximation should be used. But, as it was shown in [3], the evaluation of acoustic pressure accounting for the second order terms can be reached applying the velocity field potential obtained from the solution of a linear problem of an incident wave scattering on an target.

To clarify the problem statement even further, it worth to mention here that velocity of the translational movement of the spherical particle is supposed to be much lower than the velocity of the incident acoustic wave. Therefore, the particle displacement is considered to be quite small. In fact, solution of the problem is derived for the immobile suspended rigid spherical particle in the present paper. Accounting for the translational motion of the target results in the non-linear problem formulation and essentially complicates solution procedure.

The linear problem formulation chosen provides relatively straightforward way to derive the exact analytical solution for the velocity potential determination. At this, the boundary conditions both on the spherical surface of the particle and on the cylindrical surface of the tube are satisfied exactly.

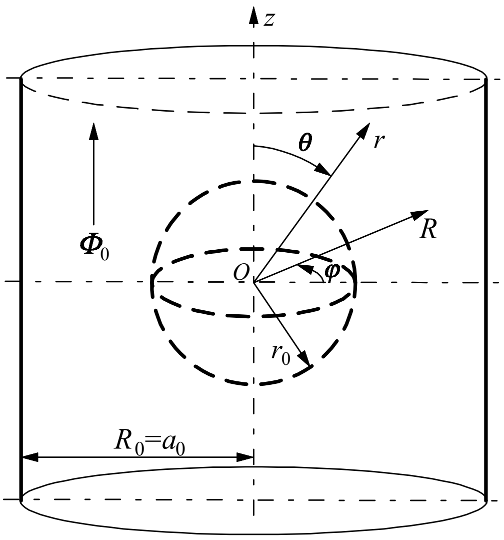


Fig. 1. Problem scheme: rigid spherical particle placed in a fluid-filled cylindrical tube

Linear approximation problem statement for the first step mentioned above, i. e,. the problem of determination of the fluid velocity potential field in a tube is described below. The procedure adopted for this aim corresponds to the approach developed in paper [28]. It is assumed that ideal compressible fluid of density ρ fills a circular cylindrical tube of radius R_0 bounded by the rigid walls. Velocity of the plane sound wave propagating in the fluid is c_0. There is an immobile spherical particle of the radius r_0 in the tube placed so that geometrical centre of it is located on the tube axis exactly. Cylindrical coordinate system (R, z, φ) is introduced in such a manner that its origin O coincides with the particle center and its axis Oz runs along the tube axis (see Fig. 1). Another coordinate system (the spherical one) (r, θ, φ) is assumed to be attached to the particle in a way shown in Fig. 1.

In this configuration, the steady wave process is under consideration. The plane harmonic wave propagating in a positive direction of the Oz axis in the tube is govern by a potential Φ_0

$$\Phi_0 = A \exp\left[i(kz - \omega t)\right], \tag{1}$$

where A is the amplitude; $k = \omega / c_0$ is the wave number; ω is the angular frequency; t is the time. The required potential Φ of the wave field is a solution of the linear wave equation

$$\Delta\Phi - \frac{1}{c_0^2}\frac{\partial^2\Phi}{\partial t^2} = 0. \tag{2}$$

In the Eq. (2) Δ is used for Laplacian operator.

Velocity of the fluid and the pressure can be determined with making use of the wave potential by the following formulas:

$$\mathbf{v} = \operatorname{grad}\Phi, \qquad p = -\rho_0 \frac{\partial \Phi}{\partial t}.$$

Let us introduce the dimensionless variables

$$\begin{gathered} \bar{R} = \frac{R}{R_0}, \qquad \bar{r} = \frac{r}{R_0}, \qquad \bar{t} = \frac{tc_0}{R_0}, \qquad \bar{\omega} = \frac{\omega R_0}{c_0}, \\ \bar{\mathbf{v}} = \frac{\mathbf{v}}{c_0}, \qquad \bar{\Phi} = \frac{\Phi}{R_0 c_0}, \qquad \bar{p} = \frac{p}{\rho_0 c_0^2}, \qquad \bar{z} = \frac{z}{R_0}. \end{gathered} \tag{3}$$

Further on the dimensionless notations will be used only. Hence, the lines over the variables will be omitted. Adoption of these conventions on the notations yields the relation (1) and (2) in the form

$$\Phi_0 = A \exp\left[i\omega(z-t)\right]; \tag{4}$$

$$\Delta\Phi + \omega^2\Phi = 0. \tag{5}$$

Interference of a primary wave and the waves scattered on the spherical particle as well as waves reflected from the internal surface forms the wave field in the cavity. From the mathematical point of view, determining of the wave field potentials in the cavity is reduced to solution of a linear problem of the acoustic wave (4) diffraction on the rigid walls of the cavity and on the rigid sphere surface, i. e,. to finding a solutions of the linear wave equation (5) accompanied by the correspondent boundary conditions, which governs the velocity field. The following conditions on the boundaries should be met: - on the tube walls

$$v_R\big|_{R=1} = \left.\frac{\partial\Phi}{\partial R}\right|_{R=1} = 0, \tag{6}$$

- on the sphere surface

$$v_r\big|_{r=r_0} = \left.\frac{\partial\Phi}{\partial r}\right|_{r=r_0} = 0, \tag{7}$$

- Sommerfeld radiation condition for infinitely distant points

$$\lim_{r\to\infty} r\left(\frac{d\Phi}{dr} - i\omega\Phi\right) = 0, \qquad \lim_{r\to\infty}\Phi = 0. \tag{8}$$

Besides, as the fluid is confined within the tube, the wave disturbances should be limited

$$\Phi \to \mathrm{const} \quad \text{if} \quad R \to 0. \tag{9}$$

Eq. (5) along with boundary conditions Eqs. (6)–(9) form the problem statement of interest.

3. WAVE FIELD POTENTIAL CONSTRUCTION

Problems of determination of the radiation forces acting on the obstacles in an ideal fluid are usually formulated with respect to the Lagrange coordinate system. Thus, the radiation pressure is defined as time-averaged value of the acoustic pressure over the obstacle surface. This approach proves to be adequate if, at determining of the acoustic pressure in a fluid, the deviation of the pressure from the harmonic law in time domain is taken into account in the obstacle vicinity [1–3]. Let us emphasize that linear approximation is insufficient for the radiation pressure evaluation because having the sound pressure as periodic function of time and averaging it over the wave field oscillation period leads to zero value of the force as it was shown in [27]. Therefore the quadratic terms in a wave equation should be preserved as well for correct estimation of the sound radiation force. In other words, the second order approximation should be used. It was previously shown in [3] that evaluation of acoustic pressure accounting for the terms of the second order of magnitude can be done using the velocity field potential obtained from the solution of a linear problem of a primary wave scattering on an obstacle. The same approach is adopted here as well.

3.1. Rigid tube wall

The boundary problem addressed comprise the wave equation (5) accompanied by the boundary conditions (6)–(9). It is formulated over the domain $R \leq 1$, $r \geq r_0$, $-\infty < z < \infty$, where r_0 is a sphere radius over the cylinder radius ratio. The complexity of the problem lies primarily in the necessity to satisfy the boundary conditions specified on the surfaces of different geometrical shape (cylindrical, spherical), on a cylinder axis as well as on the infinity.

The wave potential Φ is constructed as a superposition of three components

$$\Phi = \Phi_0 + \Phi_{\rm sph} + \Phi_{\rm cyl}, \tag{10}$$

where$\Phi_{\rm sph}$ is a potential of the wave reflected by the spherical particle, $\Phi_{\rm cyl}$ is a potential of the wave reflected from tube wall. The method of separation of variables in matching coordinate frames (spherical and cylindrical) is applied to build the potentials $\Phi_{\rm sph}$ and $\Phi_{\rm cyl}$. Thereby, the class of solutions $\Phi_{\rm sph}$ is represented in the form of a generalized Fourier series expansion with respect to spherical wave functions

$$\Phi_{\rm sph}(r,\theta) = \sum_{n=0}^{\infty} A_n h_n(\omega r) P_n(\cos\theta), \tag{11}$$

meanwhile the class of solutions $\Phi_{\rm cyl}$ is chosen in the form of an integral

$$\Phi_{\rm cyl}(\rho, z) = \int_{-\infty}^{\infty} B(\xi) J_0\left(\sqrt{\omega^2-\xi^2}R\right) e^{i\xi z} d\xi. \tag{12}$$

Here $h_n(x)$ are the first kind spherical Hankel functions, $P_n(x)$ are the Legendre polynomials; $J_0(x)$ is a zero order cylindrical Bessel function; A_n are unknown constants to be found; $B(\xi)$ is an unknown distribution density function; ξ is a separation constant. In expressions (11), (12) and further on, the multiplier $\exp(-i\omega t)$ is omitted.

To chose the relevant solution from the set of all possible solutions of the Eq. (5), the boundary conditions are used. Conditions (8) and (9) are satisfied directly if the potentials are chosen in the form (11) and (12) correspondingly.

Components Φ_0, $\Phi_{\rm sph}$ and $\Phi_{\rm cyl}$ of the general potential Φ specified by the expressions (4), (11), (12) and (10) respectively are referred to the different coordinate systems related to the spherical particle and cylindrical tube. To meet the boundary conditions (7) on the sphere surface, it is necessary to derive the representation of the general potential Φ (10) (i. e,. its components Φ_0 and $\Phi_{\rm cyl}$) in the spherical coordinate system (r, θ, φ) (see Fig. 1) while it is necessary to express the general potential Φ (i. e,. its components Φ_0 and $\Phi_{\rm sph}$) in the cylindrical coordinate frame (R, z, φ) shown in Fig. 1 to satisfy the boundary conditions (6) on the cylindrical tube surface [29]. In doing so, the cylindrical wave functions can be represented through spherical wave functions and vice versa

$$h_n(\omega r) P_n(\cos\theta) = \frac{i^{-n}}{2\omega} \int_{-\infty}^{\infty} P_n\left(\frac{\xi}{\omega}\right) H_0\left(\sqrt{\omega^2-\xi^2}R\right) e^{i\xi z} d\xi; \tag{13}$$

$$e^{i\xi z}J_0\left(\sqrt{\omega^2-\xi^2}R\right)=\sum_{n=0}^{\infty}i^n(2n+1)P_n\left(\frac{\xi}{\omega}\right)j_n(\omega r)P_n(\cos\theta). \tag{14}$$

In the expressions (13) and (14), $H_0(x)$ is a first kind zero order cylindrical Hankel function; $j_n(x)$ is spherical Bessel function of the n-th order. Using the relation (13) yields the potential Φ_{sph} in the coordinate frame related to the cylinder as follows

$$\Phi_{\text{sph}}(R,z)=\int_{-\infty}^{\infty}A(\xi)H_0\left(\sqrt{\omega^2-\xi^2}R\right)e^{i\xi z}d\xi,\qquad A(\xi)=\frac{1}{2\omega}\sum_{n=0}^{\infty}A_n i^{-n}P_n\left(\frac{\xi}{\omega}\right). \tag{15}$$

The expression of the potential in the spherical coordinates is specified by Eq. (11) already.

In the same way, application of the relations (14) yields the potential Φ_{cyl} in the coordinate system related to the sphere in the form

$$\Phi_{\text{cyl}}(r,\theta)=\sum_{n=0}^{\infty}B_n j_n(\omega r)P_n(\cos\theta),\qquad B_n=i^n(2n+1)\int_{-\infty}^{\infty}B(\xi)P_n\left(\frac{\xi}{\omega}\right)d\xi. \tag{16}$$

Again, the expression of the potential in the cylindrical coordinates is already specified by Eq. (12). Potential Φ_0 of the primary plane wave propagating along the cylindrical tube is also expressed in the form of a generalized Fourier series expansion with respect to spherical wave functions [30]

$$\Phi_0(r,\theta)=\sum_{n=0}^{\infty}Ai^n(2n+1)j_n(\omega r)P_n(\cos\theta). \tag{17}$$

Combining relations (4), (12) and (15) yields the expression for the total wave field potential Φ introduced by Eq. (10) referred to the cylindrical coordinate system in the following form:

$$\Phi(R,z)=Ae^{i\omega z}+\int_{-\infty}^{\infty}A(\xi)H_0\left(\sqrt{\omega^2-\xi^2}R\right)e^{i\xi z}d\xi+\int_{-\infty}^{\infty}B(\xi)J_0\left(\sqrt{\omega^2-\xi^2}R\right)e^{i\xi z}d\xi. \tag{18}$$

Extraction from Eq. (18) of those solutions which satisfy the boundary conditions (6) gives

$$\int_{-\infty}^{\infty}\left\{\left[A(\xi)H_1\left(\sqrt{\omega^2-\xi^2}\right)+B(\xi)J_1\left(\sqrt{\omega^2-\xi^2}\right)\right]\sqrt{\omega^2-\xi^2}e^{i\xi z}\right\}dz=0. \tag{19}$$

In view of the expression (15) for $A(\xi)$, Eq. (19) yields the formula allowing to express the function $B(\xi)$ through the unknown constants A_n

$$B(\xi)=-\frac{1}{2\omega}\frac{H_1\left(\sqrt{\omega^2-\xi^2}\right)}{J_1\left(\sqrt{\omega^2-\xi^2}\right)}\sum_{n=0}^{\infty}A_n i^{-n}P_n\left(\frac{\xi}{\omega}\right). \tag{20}$$

With taking account for relations (11), (16) and (17), the total wave field potential specified by Eq. (10) in the spherical coordinate system (r, θ, φ) takes the form:

$$\Phi(r,\theta) = \sum_{n=0}^{\infty} \{ [A i^n (2n+1) j_n(\omega r) + A_n h_n(\omega r) + B_n j_n(\omega r)] \, P_n(\cos\theta) \} \tag{21}$$

As before, the multiplier $\exp(-i\omega t)$ is omitted in Eqs. (15)–(18) and (21). Making use of Eq. (21), solutions that satisfy the boundary conditions (7) yield the equation

$$\sum_{n=0}^{\infty} \{ [A_n h_n'(\omega r_0) + B_n j_n'(\omega r_0) + A i^n (2n+1) j_n'(\omega r_0)] \, P_n(\cos\theta) \} = 0. \tag{22}$$

Taking account of expression (16) for B_n and (20) for $B(\xi)$ and making use of the orthogonality of the Legendre polynomials, Eq. (22) yields the infinite system of the algebraic equations for determination of coefficients A_n

$$A_n - \frac{(2n+1)}{2\omega} \frac{j_n'(\omega r_0)}{h_n'(\omega r_0)} \sum_{m=0}^{\infty} i^{n-m} q_{mn} A_m = -A i^n (2n+1) \frac{j_n'(\omega r_0)}{h_n'(\omega r_0)}, \qquad n = 0, 1, 2, \ldots \tag{23}$$

Coefficients q_{mn} in the Eq. (23) are of the form

$$q_{mn} = 2 \int_0^{\infty} \frac{H_1\left(\sqrt{\omega^2 - \xi^2}\right)}{J_1\left(\sqrt{\omega^2 - \xi^2}\right)} P_n\left(\frac{\xi}{\omega}\right) P_n\left(\frac{\xi}{\omega}\right) d\xi. \tag{24}$$

If sum of indices $n + m$ is odd then coefficients q_{mn} are equal to zero. The infinite system of the algebraic equations (23) possesses the completely continuous operator in a Hilbert coordinate space l^2 [31] and its right hand parts meet the following condition

$$\sum_{n=0}^{\infty} \left| A (2n+1)^2 \frac{j_n'(\omega r_0)}{h_n'(\omega r_0)} \right| < \infty.$$

Hence, the system of equations (23) has the only solution $\{A_n\}$, $n = 0, 1, 2, \ldots$ which satisfies condition

$$\sum_{n=0}^{\infty} |A_n|^2 < \infty.$$

This solution can be found by a reduction method.

Thus, determination of the total wave field potential Φ from Eq. (10) is formally completed by the evaluation of coefficients A_n of the truncated system of algebraic equations (23), meanwhile the function $B(\xi)$ in the potential Φ_{cyl} from Eq. (12) is governed by Eq (20). Required accuracy of evaluation is secured by the comparison of results obtained for sequentially increasing number of the equations of the truncated system.

3.2. Elastic tube wall

Let us consider the case of elastic tube now. As it was mentioned above, the accounting for compliance of the bounding walls is an intricate task. When the properties of the fluid and of the tube are uniform along it, the linear theory leads to the same one-dimensional wave equation as for the plane sound waves in the unbound medium, but with the value of the wave velocity affected by the distensibility of the tube [26].

The problem configuration and the domain are the same as for the case of the rigid tube wall (see Fig. 1 again). This time the tube radius is not constant due to tube wall elasticity and is dnoted by $a = a(t)$. Let us investigate the problem for the cylindrical tube of the radius $a_0 = R_0$ under zero pressure and of the wall thickness h. The Young modulus and Poisson ratio of the tube material are denoted by E and ν respectively. The tube is filled with an ideal compressible fluid of the density ρ_0 and sound velocity c_0. The rigid spherical particle of a radius r_0 is placed on the tube axis. The same cylindrical (R, z, φ) and spherical (r, θ, φ) coordinate frames are introduced as shown in Fig. 1. The considered steady process of plane wave propagation along the Oz axis is described by the potential (1). According to the approach adopted, the potential is the solution of the Eq. (2) written in the form

$$\Delta\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = 0.$$

Let us emphasize here that the only difference with equation (2) is that c is the plane wave velocity in the fluid filling the elastic tube which differs from the sound velocity c_0 in the infinite space. It can be expressed as follows [26]

$$c = \frac{c_0}{\sqrt{1 + 2R_0/(hE'\beta_0)}}, \tag{25}$$

where $E' = E/(1-\nu^2)$; $\beta_0 = 1/(\rho c_0^2)$ is the compressibility of the fluid; $R_0 = a_0$ is the tube radius under zero pressure induced by the sound wave. In the linear theory case, velocity of the fluid and the pressure can be determined with making use of the wave potential (1) by the following formulas:

$$\mathbf{v} = \operatorname{grad}\Phi, \qquad p = -\rho_0\frac{\partial\Phi}{\partial t}. \tag{26}$$

Convention on the dimensionless quantities notation adopted in the section 2 yields the same equations (4) and (5) while expressions for the velocity and pressure take the form

$$\mathbf{v} = \operatorname{grad}\Phi, \qquad p = -\frac{\partial\Phi}{\partial t}; \tag{27}$$

where additional dimensionless variables are introduced

$$\bar{a} = \frac{a}{R_0}; \qquad \bar{h} = \frac{h}{R_0}; \qquad \bar{a}_0 = 1.$$

As for the rigid spherical particle, total wave field potential is a superposition of the potentials of the incident wave and the waves reflected by the spherical particle and the cylindrical tube wall (see expression (10)). As a result, the potentials determination is reduced to solution of a linear problem of the acoustic wave (4) diffraction on the elastic walls of the tube and on the rigid sphere surface, i. e,. to finding a solutions of the linear wave equation (5) accompanied by the correspondent boundary conditions. Let us formulate the conditions on the boundaries that should be met.

Normal component of the fluid velocity is equal to zero on the surface of the rigid sphere. This condition is already written as the expression (7).

On the internal surface of the elastic tube, the radial component of the fluid velocity $\partial\Phi/\partial R$ is equal to the radial velocity of the tube wall $\dot{a}(t)$, where $a(t)$ is the internal radius of the wall in a time moment t. In the case of the linear theory, it can be written as follows

$$\left.\frac{\partial\Phi}{\partial R}\right|_{R=a_0} = \dot{a}(t), \tag{28}$$

because any difference in values between $\partial\Phi/\partial R$; at $R = a(t)$ and $R = a_0$ is less than

$$|a(t) - a_0| \max \left|\frac{\partial^2\Phi}{\partial R^2}\right|,$$

and, therefore, it can be neglected as a product of the two small quantities [26].

Potential of the waves scattered by the sphere, Φ_{sph}, should met the Sommerfeld radiation condition (8) for infinitely distant points, while potential of the waves reflected by the cylindrical tube surface, Φ_{cyl}, has to satisfy the boundary condition (9).

Construction of the wave potentials resembles the procedure described above for the case of rigid tube wall. Applying the method of separation of variables, expressions for potentials Φ_{sph} and Φ_{cyl} are taken in the form (11) and (12) correspondingly. Once again, the solutions satisfying the boundary conditions (7) and (28) should be chosen from the set (11) and (12) of all possible solutions of equation (5). It worth mentioning that solutions (11) and (12) were constructed to satisfy the boundary conditions (8) and (9) in the infinity and tube axis automatically. Following the procedure discussed in details in the previous section and repeating mathematical manipulations resulting in the formulae (13)–(21), one can derive the expression for the total wave field potential introduced by Eqs. (18) and (21) referred to the cylindrical and spherical coordinate system respectively. The radial velocity of the internal wall surface oscillations induced by the fluid pressure is taken account for by the boundary condition (28). Radial motion of the surface is caused by the time variation of the circumferential force which stretches the tube wall in the transverse direction resizing its radius.

Let us select a section of the tube of the length $l = 1$ (the wall thickness is h). One can easily show that the radial displacement of the wall points $u_R = a(t) - a_0$ is determined by the dimensionless expression

$$u_R = \frac{p}{hE'}. \tag{29}$$

Thus the boundary condition (28) can be rewritten as follows

$$\left.\frac{\partial\Phi}{\partial R}\right|_{R=a_0} = \frac{1}{hE'}\frac{dp}{dt}. \tag{30}$$

Using potential (18) along with (10), one can write condition (30) for $z = 0$ in the form

$$\int_{-\infty}^{\infty} \left[A(\xi) H_1\left(\sqrt{\omega^2-\xi^2}\right) + B(\xi) J_1\left(\sqrt{\omega^2-\xi^2}\right)\right] \sqrt{\omega^2-\xi^2} dz = \\ = \frac{\omega^2}{hE'} \int_{-\infty}^{\infty} \left[A \frac{1}{\pi(1+\xi^2)} + A(\xi) H_0\left(\sqrt{\omega^2-\xi^2}\right) + B(\xi) J_0\left(\sqrt{\omega^2-\xi^2}\right)\right] d\xi. \tag{31}$$

In (31), the following expression is used

$$\pi = \int_{-\infty}^{\infty} \frac{d\xi}{1+\xi^2}. \tag{32}$$

Let us introduce the following notations

$$C(\omega,\xi) = H_1\left(\sqrt{\omega^2-\xi^2}\right)\sqrt{\omega^2-\xi^2} - H_0\left(\sqrt{\omega^2-\xi^2}\right)\frac{\omega^2}{hE'}, \\ D(\omega,\xi) = J_1\left(\sqrt{\omega^2-\xi^2}\right)\sqrt{\omega^2-\xi^2} - J_0\left(\sqrt{\omega^2-\xi^2}\right)\frac{\omega}{hE'}. \tag{33}$$

Then, applying for (31) and making use of (33), the boundary condition (30) yields

$$\int_{-\infty}^{\infty} \left[A(\xi) C(\omega,\xi) + B(\xi) D(\omega,\xi) - A\frac{\omega^2}{\pi hE'(1+\xi^2)}\right] d\xi = 0. \tag{34}$$

The boundary condition on the surface of the rigid sphere is unaffected and is preserved the form (22).

Making use of the orthogonality of the Legendre polynomials, Eq. (22) yields the infinite system of the algebraic equations for determination of coefficients A_n and B_n

$$A_n h_n'(\omega r_0) + B_n j_n'(\omega r_0) = -A(2n+1) i^n j_n'(\omega r_0), \qquad n = 0, 1, 2 \ldots. \tag{35}$$

Coefficients B_n is expressible in terms of the A_n. Making use of expressions (16), (34) and (15), one can derive

$$B_n = -\frac{2n+1}{2\omega} \sum_{m=0}^{\infty} i^{n-m} q_{mn} A_m + \frac{2n+1}{\pi} i^n G(\omega,\xi). \tag{36}$$

In the formula (36), the following expressions are used

$$G(\omega,\xi) = \int_{-\infty}^{\infty} \frac{A\omega^2}{hE' D(\omega,\xi)(1+\xi^2)} P_n\left(\frac{\xi}{\omega}\right) d\xi, \tag{37}$$

$$q_{mn} = 2\int_{0}^{\infty} \frac{C(\omega,\xi)}{D(\omega,\xi)} P_m\left(\frac{\xi}{\omega}\right) P_n\left(\frac{\xi}{\omega}\right) d\xi. \tag{38}$$

From (35), taking account of the expression (36) for the coefficients B_n, the infinite system of the algebraic equations for determination of coefficients A_n

$$A_n - \frac{(2n+1)}{2\omega}\frac{j_n'(\omega r_0)}{h_n'(\omega r_0)}\sum_{m=0}^{\infty} i^{n-m} q_{mn} A_m = -(2n+1) i^n \left[A + \frac{G(\omega,\xi)}{\pi}\right]\frac{j_n'(\omega r_0)}{h_n'(\omega r_0)}, \quad n = 0, 1, 2\ldots \tag{39}$$

which is analogous to the system (23) derived for the case of rigid tube wall.

The system (39) can be solved in the same way as the system (23). The technique is described in section 3.2.

4. ACOUSTIC RADIATION FORCE DETERMINATION

Acoustic radiation force affecting spherical particle submerged in a fluid is equal to the hydrodynamic force acting on a spherical particle averaged over the period of an incident wave. In the configuration under study, the hydrodynamic force is directed along the Oz axis because the fluid velocity field in the tube is symmetrical with respect to this axis. This force can be calculated as an integral of the pressure p over the particle surface

$$F_z = -2\pi r_0^2 \int_0^{\pi} p \sin\theta \cos\theta d\theta, \tag{40}$$

where the real part of expression for the pressure should be used. Pressure of the acoustic field on the spherical particle surface has to be computed with the accuracy up to the terms of the second order of magnitude. The expression for pressure from [3] in the dimensionless form is used

$$p = -\frac{\partial \Phi}{\partial t} - \frac{1}{2r_0^2}\left(\frac{\partial \Phi}{\partial \theta}\right)^2 + \frac{1}{2}\left(\frac{\partial \Phi}{\partial t}\right)^2. \tag{41}$$

Contribution of the first term in the right hand side of Eq. (41) is zero because it varies sinusoidally in time [20]. This term will therefore not be taken into consideration any further. The second summand is equal to zero as well for the immobile particle because it is defined by the radial component of a fluid velocity on a surface of the spherical particle. Real part of the total wave field potential (21) is of the form

$$\mathbf{Re}\,\Phi = \sum_{n=0}^{\infty}(S_n \cos\omega t + R_n \sin\omega t) P_n(\cos\theta), \tag{42}$$

where

$$\begin{aligned}
&S_n = K_n + M_n + G_n, && R_n = L_n + T_n + N_n;\\
&K_n = (-1)^n A(4n+1) j_{2n}(\omega r_0); && L_n = (-1)^n A(4n+3) j_{2n+1}(\omega r_0);\\
&M_n = \mathbf{Re}\,(A_n) j_n(\omega r_0) - \mathbf{Im}\,(A_n) y_n(\omega r_0); && N_n = \mathbf{Re}\,(A_n) y_n(\omega r_0) + \mathbf{Im}\,(A_n) j_n(\omega r_0);\\
&G_n = \mathbf{Re}\,(B_n) j_n(\omega r_0); && T_n = \mathbf{Im}\,(B_n) j_n(\omega r_0).
\end{aligned} \tag{43}$$

Then, making use of Eqs. (40)–(43) and omitting a symbol **Re** for brevity, the pressure p and force F_z can be calculated. Finally, taking account of the reasoning concerning Eq. (41) mentioned, the expression for the pressure p takes the form

$$p = -\frac{1}{2r_0^2}\left(\frac{\partial\Phi}{\partial\theta}\right)^2 + \frac{1}{2}\left(\frac{\partial\Phi}{\partial t}\right)^2. \tag{44}$$

Contribution of the first term from (44) is determined by an integral

$$F_z^{(1)} = \pi\int\limits_0^\pi \left(\frac{\partial\Phi}{\partial\theta}\right)^2 \sin\theta\cos\theta d\theta. \tag{45}$$

Substituting (42) into Eq. (45) and making use of the relations

$$\int\limits_{-1}^{1}\frac{dP_n(\mu)}{d\mu}\frac{dP_m(\mu)}{d\mu}\mu(1-\mu^2)d\mu = \begin{cases} \dfrac{2n(n+1)(n+2)}{(2n+1)(2n+3)}, & \text{if } m = n+1, \\ 0, & \text{if } m \neq n+1, \end{cases} \quad \text{and} \quad \mu = \cos\theta,$$

yields the expression for the contribution of the first term from Eq. (44) to the total value of the hydrodynamic force F_z

$$F_z^{(1)} = 2\pi\sum_{n=0}^{\infty}\frac{2n(n+1)(n+2)}{(2n+1)(2n+3)}\left[R_nR_{n+1}\sin^2(\omega t) + S_nS_{n+1}\cos^2(\omega t)\right]. \tag{46}$$

The terms resulting in nonzero values being averaged over the period are preserved in expression (46) only.

Averaging of (46) over the incident wave period yields the contribution of the first term from the expression for pressure (44) to the total value of the radiation force

$$\left\langle F_z^{(1)}\right\rangle = 2\pi\sum_{n=0}^{\infty}\frac{n(n+1)(n+2)}{(2n+1)(2n+3)}(R_nR_{n+1} + S_nS_{n+1}). \tag{47}$$

The contribution of second summand from (44) is governed by an integral

$$F_z^{(2)} = -\pi r_0^2\int\limits_0^\pi \left(\frac{\partial\Phi}{\partial t}\right)^2 \sin\theta\cos\theta d\theta. \tag{48}$$

Making use of Eq. (42) and relation

$$\int\limits_{-1}^{1} P_n(\mu)P_m(\mu)\mu d\mu = \begin{cases} \dfrac{2(n+1)}{(2n+1)(2n+3)}, & \text{if } m = n+1, \\ 0, & \text{if } m \neq n+1, \end{cases} \tag{49}$$

integration of Eq. (48) yields the formula for estimation of the second term contribution to the force F_z

$$F_z^{(2)} = -4\pi r_0^2\omega^2\sum_{n=0}^{\infty}\frac{(n+1)}{(2n+1)(2n+3)}\left[R_nR_{n+1}\sin^2(\omega t) + S_nS_{n+1}\cos^2(\omega t)\right]. \tag{50}$$

Repeating the reasoning on zero sinusoidal terms contribution being averaged over the period of incident wave yields the contribution of the second term from (44) to the total value of the radiation force as follows

$$\left\langle F_z^{(2)} \right\rangle = -2\pi r_0^2 \omega^2 \sum_{n=0}^{\infty} \frac{(n+1)}{(2n+1)(2n+3)} (R_n R_{n+1} + S_n S_{n+1}). \tag{51}$$

Summation of expressions (47) and (51) gives the final formula for the acoustic radiation force affecting the rigid spherical particle submerged in a fluid filled tube with rigid or elastic walls

$$\langle F_z \rangle = 2\pi \sum_{n=0}^{\infty} \frac{n+1}{(2n+1)(2n+3)} \left\{ (R_n R_{n+1} + S_n S_{n+1}) \left[n(n+2) - (\omega d)^2 \right] \right\}. \tag{52}$$

The formulae obtained through the mathematical manipulations above show the expressions for the acoustic radiation force are quite similar for the wall types considered. However the different boundary conditions on the rigid and elastic tube wall written in the form (19) and (34) respectively result in different systems ((23) for the rigid and (39) for the elastic wall) for determination of the unknown coefficients of the wave field potential expansion.

5. NUMERICAL RESULTS

As an example of the developed theory application, investigation of the radiation force influencing a rigid spherical particle located in a fluid-filled circular cylindrical tube is carried out for the case of rigid tube wall. Calculations are performed numerically wit making use of equation (44) for pressure and (52) for the ARF itself. The following characteristics of a compressible fluid and sound radiation power are used: a sound velocity is $c_0 = 1500$ m/sec, density is $\rho_0 = 1000$ kg/m^3, an energy flux density is $I = 175.5$ Wt/m^2, the wave length is $\lambda = 0.038$ m and velocity of particles of a fluid is 0.015 m/sec. These parameters correspond to moderate radiated power. Dimensionless amplitude A in the expression (4) is chosen to be equal to $3 \cdot 10^{-6}$ for the case under consideration.

The calculation algorithm is organized as a two stage procedure. At the first stage, the determination of the velocity field potential according to the Eq. (21) is carried out. For this aim, calculation of the coefficients A_n and B_n from the truncated system of the algebraic equations (23) with account of the expressions (16), (20) and (24) is carried out with the preset accuracy of 10^{-6} which is secured by comparison of the evaluation results for sequentially increasing number of the equations in the truncated system. As soon as the A_n and B_n are found, the absolute value of the linear hydrodynamic acoustic pressure is calculated with making use of the expression

$$p = -\rho_0 \frac{\partial \Phi}{\partial t} = i\omega c_0^{-2} \Phi.$$

At the second stage, the ARF itself is calculated making use of (44) and (52).

To characterize the pressure distribution in the vicinity of the spherical particle, the dimensionless amplitude of pressure p is calculated vs the excitation frequency ω for three points of the tube cross-section passing through the center of the particle. The results are

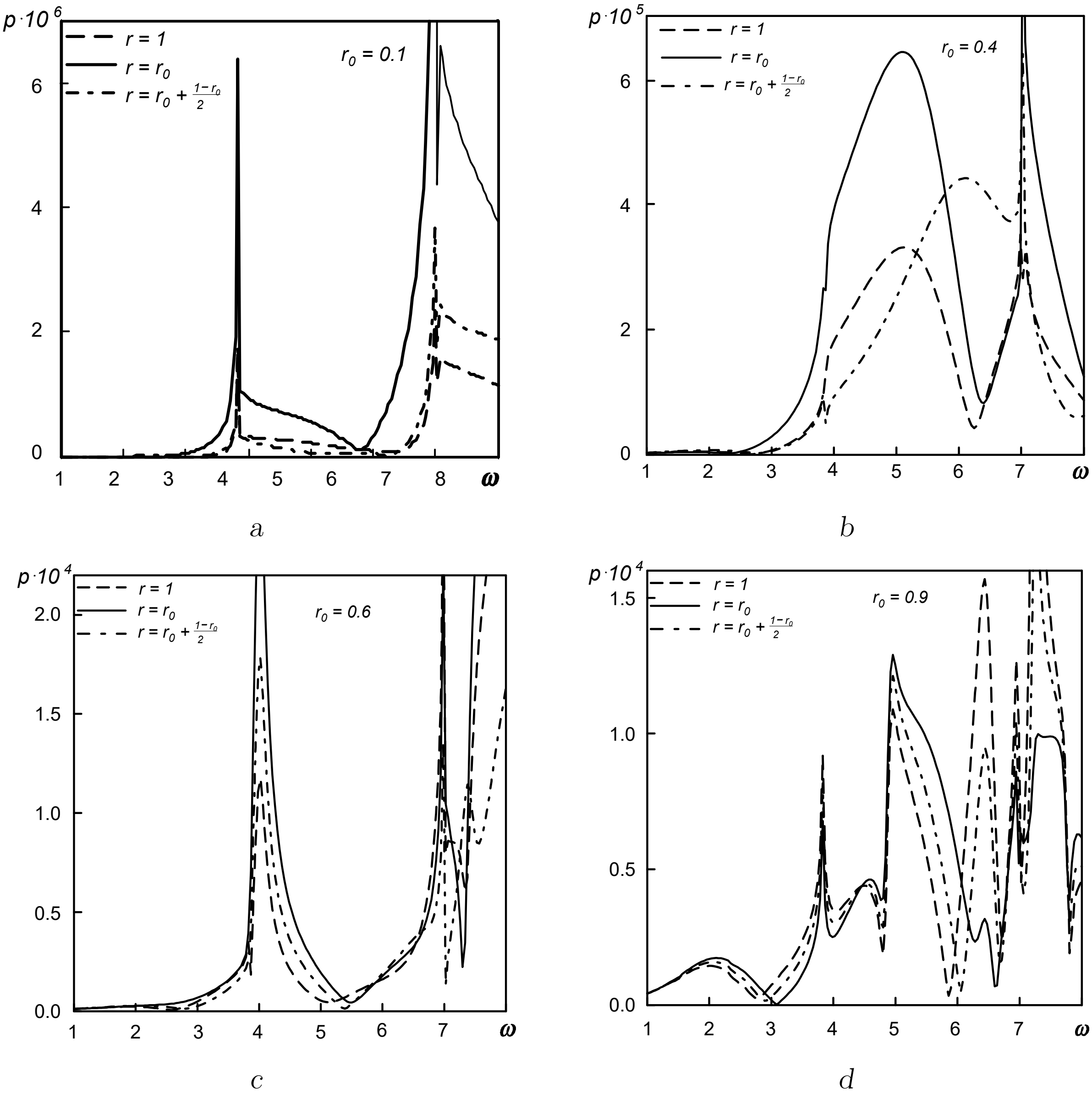


Fig. 2. Pressure vs incident wave frequency dependence
for three points of the central sphere cross-section:
solid — point on the sphere surface; *dashed* — point on the tube surface;
dash-dot — midpoint between surfaces of the tube and sphere

shown in Fig. 2. Solid line corresponds to pressure at the point on the spherical particle surface ($r = r_0$), dashed line stands for the pressure on the tube surface ($R = 1$) and dash-dot line illustrates the pressure variation at midpoint between surfaces of sphere and tube. Figs. 2a, b and c plotted for normalized sphere radii 0.4, 0.6 and 0.9 respectively. Two peaks in pressure values are observed approximately in the vicinity of frequencies 3.8 and 7.0 in Fig. 2. In Fig. 2a ($r_0 = 0.4$), the first peak is quite small. It is due to the narrowness of the region itself and the fact that calculation with fixed frequency step provides quite coarse set of calculation points to catch the exact peak value. Amount of the peaks increases as the particle radius grows. As it was shown in [4], the frequencies of these new emerging peaks correspond to the natural frequencies of the fluid ring formed by the cross-section of the rigid tube passing through the center of the sphere.

Having the pressure calculated, the radiation force can be determined as a function of frequency. From the practical point of view, the influence of the rigid tube surface on the ARF is of particular interest because the diffraction on the incident wave on surface of both the particle and the tube wall can lead to complex unpredictable effects. Thus, the ARF effect on the rigid spherical particle located in a fluid-filled cylindrical tube can be clarified by comparison with the results obtained for the particle placed in an unbounded fluid. In order to do this, the statement of the problem of solution determination for the wave equation (5) for the incident wave (4) is developed. For the case of unbounded fluid space, the boundary conditions (8) and (9) should be satisfied only. Solution of this problem is the form (20) exactly when all coefficients B_n are equal to zero. Thus coefficients A_n can be found as follows

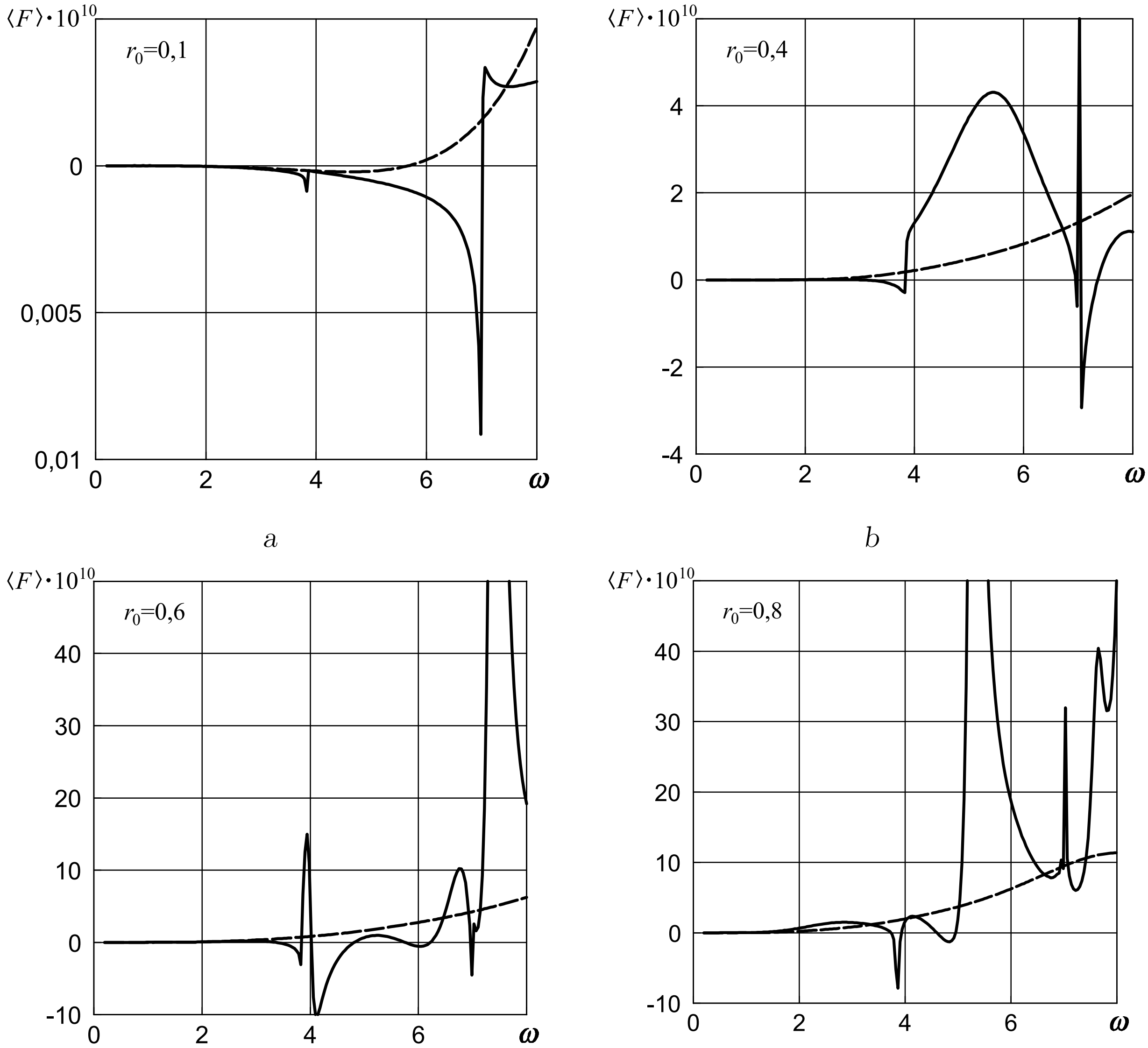
$$A_n = -i^n(2n+1)\frac{j'_n(\omega r_0)}{h'_n(\omega r_0)}, \qquad n = 0, 1, 2, \ldots$$

Dependencies of the dimensionless values of the radiation force on the incident wave frequency are presented in Fig. 3a to d for $r_0 = 0.1$, 0.4, 0.6 and 0.8 respectively. All these dependencies are depicted by the solid line in the figure. Dashed lines are reserved for case of the spherical particle located in an unbounded fluid space.

In the Fig. 3a, behavior of the ARF for a ratio of spherical particle radius to the tube radius of 0.1 is illustrated. Comparison of the cases of bounded tube and unbounded fluid space shows two main differences. Contrary to the monotonic increase in the latter case, the ARF affecting a spherical particle located in the fluid-filled tube demonstrates easily recognizable maxima and can change the direction of the force. In this case, ARF acts on the particle pushing it in the direction which is opposite to the direction of the incident wave. This effect can apparently be associated with relatively weak scattering into the backward hemisphere [6]. Comparison of Figs 3b to d shows that these effects become even more prominent as the sphere radius increases. Peak values of the radiation force correspond to the natural frequencies of the fluid ring formed by the cross-section of the cavity passing through the center of the spherical particle [28].

6. CONCLUSIONS

The following conclusions can be drawn from the data calculated. Technique of calculation of the ARF acting on the rigid spherical particle placed in the fluid-filled cylindrical tube is elaborated. The cases of both rigid and elastic tube wall are considered. The technique elaborated is applicable for ARF calculation with the preset accuracy. The values of the



a *b* *c* *d*

Fig. 3. Dependence of the ARF on the incident wave frequency. Particle is placed in an unbounded fluid (dashed line) or in a cylindrical tube with rigid wall (solid line): a — $r_0/R_0 = 0.1$; b — $r_0/R_0 = 0.4$; c — $r_0/R_0 = 0.6$; d — $r_0/R_0 = 0.8$

ARF are estimated for the ranges of the incident wave frequency and the ratio between the radii of a cylindrical tube and a spherical particle.

In the unbounded fluid, direction of the ARF acting on a rigid spherical particle located in it coincides with the incident wave direction and increases monotonically as the frequency rises. Complexity of the diffraction phenomena in the vicinity of a spherical particle placed in the fluid-filled cylindrical tube with rigid or elastic wall causes complex character of the ARF dependence on the incident wave frequency.

For the particle located in the fluid-filled cylindrical tube, the plane acoustic wave propagating along the tube axis induces the ARF of the same or opposite direction depending on the incident wave frequency. In this particular case, the sharp peaks in the ARF values are observed in the vicinity of some frequencies. These maxima are attributed to the resonant effects that occur at the frequencies that are close to the natural frequencies of the liquid ring formed by the cross-section of the tube passing through the center of the rigid spherical particle.

The developed technique which is based on the exact analytical solution with making use of variables separation technique and mutual expansions of spherical wave functions over the cylindrical ones and vice versa can be easily generalized to the case a finite number of spherical particles arbitrarily located in a circular cylindrical tube with rigid or elastic wall.

ACKNOWLEDGEMENTS

Publication is based on the research provided by the grant support of the National Research Foundation of Ukraine (project No. 2020.02/0112).

REFERENCES

[1] B. O. Enflo and C. M. Hedberg, *Theory of nonlinear acoustics in fluids*. Dordrecht: Kluwer Academic Publishers, 2002.

[2] A. D. Pierce, *Acoustics an introduction to its physical principles and applications*. New York: McGraw-Hill, 1981.

[3] L. V. King, “On the acoustic radiation pressure on spheres,” *Proceedings of the Royal Society of London. Series A: Mathematical and Physical Sciences*, vol. 147, no. 861, pp. 212–240, 1934. https://doi.org/10.1098/rspa.1934.0215

[4] A. P. Zhuk, V. D. Kubenko, and Y. A. Zhuk, “Acoustic radiation force on a spherical particle in a fluid-filled cavity,” *The Journal of the Acoustical Society of America*, vol. 132, no. 4, pp. 2189–2197, 2012. https://doi.org/10.1121/1.4739440

[5] E. P. Mednikov, *Acoustic coagulation and precipitation of aerosols*. New York: Springer, 2013.

[6] F. Cai, L. Meng, C. Jiang, Y. Pan, and H. Zheng, “Computation of the acoustic radiation force using the finite-difference time-domain method,” *The Journal of the Acoustical Society of America*, vol. 128, no. 4, pp. 1617–1622, 2010. https://doi.org/10.1121/1.3474896

[7] K. A. Fisher and R. Miles, "Modeling the acoustic radiation force in microfluidic chambers," *The Journal of the Acoustical Society of America*, vol. 123, no. 4, pp. 1862–1865, 2008. https://doi.org/10.1121/1.2839140

[8] J. Wang and J. Dual, "Two-dimensional numerical simulations for the time-averaged acoustic forces acting on a rigid particle of arbitrary shape in a standing wave." *The Journal of the Acoustical Society of America*, vol. 125, no. 4 Suppl., p. 2594, 2009. https://doi.org/10.1121/1.4783863

[9] S. D. Danilov and M. A. Mironov, "Mean force on a small sphere in a sound field in a viscous fluid," *The Journal of the Acoustical Society of America*, vol. 107, no. 1, pp. 143–153, 2000. https://doi.org/10.1121/1.428346

[10] A. N. Guz and A. P. Zhuk, "On the hydrodynamic forces in an acoustic field in a viscous liquid," *Doklady Akademii Nauk SSSR*, vol. 266, no. 1, pp. 32–35, 1982.

[11] A. P. Zhuk, "Dynamics of a spherical particle near a flat liquid boundary under acoustic radiation forces," *International Applied Mechanics*, vol. 44, no. 11, pp. 1223–1232, 2008. https://doi.org/10.1007/s10778-009-0144-4

[12] R. Löfstedt and S. Putterman, "Theory of long wavelength acoustic radiation pressure," *The Journal of the Acoustical Society of America*, vol. 90, no. 4, pp. 2027–2033, 1991. https://doi.org/10.1121/1.401630

[13] J. Wu, G. Du, S. S. Work, and D. M. Warshaw, "Acoustic radiation pressure on a rigid cylinder: An analytical theory and experiments," *The Journal of the Acoustical Society of America*, vol. 87, no. 2, pp. 581–586, 1990. https://doi.org/10.1121/1.398927

[14] A. A. Doinikov, "Acoustic radiation pressure on a rigid sphere in a viscous fluid," *Proceedings of the Royal Society of London. Series A: Mathematical and Physical Sciences*, vol. 447, no. 1931, pp. 447–466, 1994. https://doi.org/10.1098/rspa.1994.0150

[15] F. E. Fox, "Sound pressure on spheres," *The Journal of the Acoustical Society of America*, vol. 12, no. 1, pp. 147–149, 1940. https://doi.org/10.1121/1.1916086

[16] J. J. Faran, "Sound scattering by solid cylinders and spheres," *The Journal of the Acoustical Society of America*, vol. 23, no. 4, pp. 405–418, 1951. https://doi.org/10.1121/1.1906780

[17] T. Hasegawa and K. Yosioka, "Acoustic-radiation force on a solid elastic sphere," *The Journal of the Acoustical Society of America*, vol. 46, no. 5B, pp. 1139–1143, 1969. https://doi.org/10.1121/1.1911832

[18] J. Shi, X. Zhang, R. Chen, and G. Zhang, "Acoustic radiation force of a solid elastic sphere immersed in a cylindrical cavity filled with ideal fluid," *Wave Motion*, vol. 80, pp. 37–46, 2018. https://doi.org/10.1016/j.wavemoti.2018.03.005

[19] T. G. Wang and C. P. Lee, *Radiation pressure and acoustic levitation*. Cham: Springer Nature Switzerland, 2024, pp. 175–203. https://doi.org/10.1007/978-3-031-58963-8_6

[20] A. N. Guz and A. P. Zhuk, "Motion of solid particles in a liquid under the action of an acoustic field: the mechanism of radiation pressure," *International Applied Mechanics*, vol. 40, no. 3, pp. 246–265, 2004. https://doi.org/10.1023/b:inam.0000031907.28520.7b

[21] A. N. Guz and A. P. Zhuk, "Dynamics of a rigid cylinder near a plane boundary in the radiation field of an acoustic wave," *Journal of Fluids and Structures*, vol. 25, no. 7, pp. 1206–1212, 2009. https://doi.org/10.1016/j.jfluidstructs.2009.06.003

[22] G. T. Silva, "An expression for the radiation force exerted by an acoustic beam with arbitrary wavefront (L)," *The Journal of the Acoustical Society of America*, vol. 130, no. 6, pp. 3541–3544, 2011. https://doi.org/10.1121/1.3652894

[23] J. Wang and J. Dual, "Theoretical and numerical calculations for the time-averaged acoustic force and torque acting on a rigid cylinder of arbitrary size in a low viscosity fluid," *The Journal of the Acoustical Society of America*, vol. 129, no. 6, pp. 3490–3501, 2011. https://doi.org/10.1121/1.3589247

[24] P. L. Marston, "Radiation force of a helicoidal Bessel beam on a sphere," *The Journal of the Acoustical Society of America*, vol. 125, no. 6, pp. 3539–3547, 2009. https://doi.org/10.1121/1.3119625

[25] A. N. Guz, A. P. Zhuk, and N. V. Gerashchenko, "On the motion of cylinder located in the vicinity of rigid plane surface in the radiation field of sound wave," *Dopovidi NAN Ukrainy*, no. 11, pp. 61–65, 1994.

[26] J. Lighthill, *Waves in fluids*. Cambridge, UK: Cambridge University Press, 1978.

[27] S. N. Rzhevkin, *A course of lectures on the theory of sound*. New York: Macmillan, 1963.

[28] V. D. Kubenko and V. V. Dzyuba, "Diffraction of a plane acoustic wave by a rigid sphere in a cylindrical cavity: An axisymmetric problem," *International Applied Mechanics*, vol. 45, no. 4, pp. 424–432, 2009. https://doi.org/10.1007/s10778-009-0195-6

[29] V. D. Kubenko, "Diffraction of steady waves at a set of spherical and cylindrical bodies in an acoustic medium," *Soviet Applied Mechanics*, vol. 23, no. 6, pp. 605–610, 1987. https://doi.org/10.1007/bf00887032

[30] A. N. Guz and V. T. Golovchan, *Diffraction of elastic waves in multiply connected bodies*. Kiev: Naukova Dumka, 1972.

[31] L. V. Kantorovich and V. I. Krylov, *Approximate methods of higher analysis*. New York: Interscience Publishers, 1958.

ЛІТЕРАТУРА

[1] Enflo B. O., Hedberg C. M. Theory of nonlinear acoustics in fluids. — Dordrecht : Kluwer Academic Publishers, 2002. — 282 p.

[2] Pierce A. D. Acoustics an introduction to its physical principles and applications. — New York : McGraw-Hill, 1981. — 123 p.

[3] King L. V. On the acoustic radiation pressure on spheres // Proceedings of the Royal Society of London. Series A: Mathematical and Physical Sciences. — 1934. — Vol. 147, no. 861. — P. 212–240.

[4] Zhuk A. P., Kubenko V. D., Zhuk Y. A. Acoustic radiation force on a spherical particle in a fluid-filled cavity // The Journal of the Acoustical Society of America. — 2012. — Vol. 132, no. 4. — P. 2189–2197.

[5] Mednikov E. P. Acoustic coagulation and precipitation of aerosols. — New York : Springer, 2013. — 192 p.

[6] Computation of the acoustic radiation force using the finite-difference time-domain method / Cai F., Meng L., Jiang C., Pan Y., and Zheng H. // The Journal of the Acoustical Society of America. — 2010. — Vol. 128, no. 4. — P. 1617–1622.

[7] Fisher K. A., Miles R. Modeling the acoustic radiation force in microfluidic chambers // The Journal of the Acoustical Society of America. — 2008. — Vol. 123, no. 4. — P. 1862–1865.

[8] Wang J., Dual J. Two-dimensional numerical simulations for the time-averaged acoustic forces acting on a rigid particle of arbitrary shape in a standing wave. // The Journal of the Acoustical Society of America. — 2009. — Vol. 125, no. 4 Suppl. — P. 2594.

[9] Danilov S. D., Mironov M. A. Mean force on a small sphere in a sound field in a viscous fluid // The Journal of the Acoustical Society of America. — 2000. — Vol. 107, no. 1. — P. 143–153.

[10] Гузь А. Н., Жук А. П. О гидродинамических силах, действующих в акустическом поле в вязкой жидкости // Доклады Академии наук СССР. — 1982. — Т. 266, № 1. — С. 32–35.

[11] Zhuk A. P. Dynamics of a spherical particle near a flat liquid boundary under acoustic radiation forces // International Applied Mechanics. — 2008. — Vol. 44, no. 11. — P. 1223–1232.

[12] Löfstedt R., Putterman S. Theory of long wavelength acoustic radiation pressure // The Journal of the Acoustical Society of America. — 1991. — Vol. 90, no. 4. — P. 2027–2033.

[13] Acoustic radiation pressure on a rigid cylinder: An analytical theory and experiments / Wu J., Du G., Work S. S., and Warshaw D. M. // The Journal of the Acoustical Society of America. — 1990. — Vol. 87, no. 2. — P. 581–586.

[14] Doinikov A. A. Acoustic radiation pressure on a rigid sphere in a viscous fluid // Proceedings of the Royal Society of London. Series A: Mathematical and Physical Sciences. — 1994. — Vol. 447, no. 1931. — P. 447–466.

[15] Fox F. E. Sound pressure on spheres // The Journal of the Acoustical Society of America. — 1940. — Vol. 12, no. 1. — P. 147–149.

[16] Faran J. J. Sound scattering by solid cylinders and spheres // The Journal of the Acoustical Society of America. — 1951. — Vol. 23, no. 4. — P. 405–418.

[17] Hasegawa T., Yosioka K. Acoustic-radiation force on a solid elastic sphere // The Journal of the Acoustical Society of America. — 1969. — Vol. 46, no. 5B. — P. 1139–1143.

[18] Acoustic radiation force of a solid elastic sphere immersed in a cylindrical cavity filled with ideal fluid / Shi J., Zhang X., Chen R., and Zhang G. // Wave Motion. — 2018. — Vol. 80. — P. 37–46.

[19] Wang T. G., Lee C. P. Radiation pressure and acoustic levitation // Nonlinear Acoustics. — Cham : Springer Nature Switzerland, 2024. — P. 175–203.

[20] Guz A. N., Zhuk A. P. Motion of solid particles in a liquid under the action of an acoustic field: the mechanism of radiation pressure // International Applied Mechanics. — 2004. — Vol. 40, no. 3. — P. 246–265.

[21] Guz A. N., Zhuk A. P. Dynamics of a rigid cylinder near a plane boundary in the radiation field of an acoustic wave // Journal of Fluids and Structures. — 2009. — Vol. 25, no. 7. — P. 1206–1212.

[22] Silva G. T. An expression for the radiation force exerted by an acoustic beam with arbitrary wavefront (L) // The Journal of the Acoustical Society of America. — 2011. — Vol. 130, no. 6. — P. 3541–3544.

[23] Wang J., Dual J. Theoretical and numerical calculations for the time-averaged acoustic force and torque acting on a rigid cylinder of arbitrary size in a low viscosity fluid // The Journal of the Acoustical Society of America. — 2011. — Vol. 129, no. 6. — P. 3490–3501.

[24] Marston P. L. Radiation force of a helicoidal Bessel beam on a sphere // The Journal of the Acoustical Society of America. — 2009. — Vol. 125, no. 6. — P. 3539–3547.

[25] Гузь О. М., Жук О. П., Геращенко М. В. Про рух циліндра біля твердої плоскої поверхні в радіаційному полі звукової хвилі // Доповіді НАН України. — 1994. — № 11. — С. 61–65.

[26] Lighthill J. Waves in fluids. — Cambridge, UK : Cambridge University Press, 1978. — 508 p.

[27] Rzhevkin S. N. A course of lectures on the theory of sound. — New York : Macmillan, 1963. — 464 p.

[28] Kubenko V. D., Dzyuba V. V. Diffraction of a plane acoustic wave by a rigid sphere in a cylindrical cavity: An axisymmetric problem // International Applied Mechanics. — 2009. — Vol. 45, no. 4. — P. 424–432.

[29] Kubenko V. D. Diffraction of steady waves at a set of spherical and cylindrical bodies in an acoustic medium // Soviet Applied Mechanics. — 1987. — Vol. 23, no. 6. — P. 605–610.

[30] Гузь А. Н., Головчан В. Т. Дифракция упругих волн в многосвязных телах. — Киев : Наукова думка, 1972. — 254 с.

[31] Kantorovich L. V., Krylov V. I. Approximate methods of higher analysis. — New York : Interscience Publishers, 1958. — 681 p.

О. П. Жук, В. Д. Кубенко, Я. О. Жук

Розрахунок акустичної радіаційної сили для жорсткої сферичної частинки розташованої в заповненій рідиною циліндричній трубці з жорсткими чи пружними стінками

Розраховано акустичний тиск та акустичну радіаційну силу (АРС), які діють на сферичну тверду частинку, занурену в рідину, що заповнює жорстку або пружну циліндричну трубку. Виведено вираз для АРС для випадку, коли падаюча хвиля є плоскою хвилею, що поширюється вздовж осі трубки. Акустичний тиск і АРС визначені як функції частоти первинної хвилі. Поставлену задачу розв'язано з використанням раніше отриманих виразів для розсіювання плоскої гармонічної хвилі на сферичному тілі. При цьому застосовано метод розділення змінних. Найскладнішою частиною проблеми є виконання граничних умов на циліндричних і сферичних поверхнях. Задля цього використано взаємні розклади сферичних хвильових функцій по циліндричних і навпаки. Точний аналітичний розв'язок отримано як розклад за спеціальною функцією. Коефіцієнти розкладу обчислюються за допомогою нескінченної системи алгебраїчних рівнянь, яка розв'язується методом редукції. Результати моделювання показали, що наявність циліндричної граничної поверхні суттєво впливає на значення АРС. Це зумовлено впливом акустичної хвилі, розсіяної циліндричною поверхнею трубки. Гострі резонансно-подібні піки виникають у тих самих місцях на залежностях АРС від частоти падаючої хвилі. Це спричинено резонансною поведінкою заповненого рідиною циліндра, тобто, наявність піків при певних дискретних значеннях частоти зумовлена ??специфічними ефектами локалізації та впливає на рух частинок у циліндричній порожнині, заповненій рідиною. Також виявлено, що залежно від частоти падіння хвилі, рух частинок може змінювати свій напрямок. Цей ефект забезпечує наукове підґрунтя для методу керування частинками, розташованими в заповнених рідиною трубках з жорсткими та пружними стінками.

КЛЮЧОВІ СЛОВА: акустична радіаційна сила, циліндрична трубка, сферична частина, ідеальна стислива рідина, плоска гармонічна хвиля