ENERGY EXTRACTION BY AN OSCILLATING SYSTEM FROM A GENERATOR OR A WAVE FIELD

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The purpose of our work is study an oscillating system and an electro-dynamical transducer, which are driven either by the amplifier or wave field. In the first case an amplifier is considered as a self-exciting system with a limited power. Electrical current produced by it is converted by the transducer into mechanical force, which leads to vibrations of the base. A mechanical oscillator is mounted on the transducer base. The influence of oscillator vibrations on the formation of the driving force leads to a number of specific effects, in particular, to the Sommerfeld –Kononenko's effect. New nonlinear effects in the coupled shaker–oscillator system are studied in details. Steady-state regimes of the constructed model are investigated by methods of the theory of dynamical systems. Regular periodic and chaotic regimes are found and studied. Expressions for supplied and consumed powers are shown and investigated for regular and chaotic regimes. The inverse problem model is also discussed. The classical results for wave power absorption by wave energy extractor as a single degree of freedom system are presented in the second considered problem. The example includes an axisymmetric buoy which oscillates and is subjected to its natural hydrostatic restoring force. Main attention is focuses on the values and expressions for the mean powers. The expression for the maximum mean power is given for the considering system.

INTRODUCTION

The coupling effect between an excitation machine and vibrational loads was found by Sommerfeld [1–3], is a universal phenomenon and a manifestation of the law of conservation of energy. A rather complete study of the Sommerfeld effect has been given in the works of Kononenko [4], so that we call these phenomena as Sommerfeld-Kononenko's effect [5-7]. As shown by Kononenko for a linear oscillator with limited excitation the characteristics of a nonlinear oscillator arise, such as the occurrence of instability regions. In view of this, in the present study, the existence of new possible characteristics is investigated for an oscillator with damping and an electro-dynamic shaker. Presence of both direct and feedback interactions between the oscillator and the shaker are main goal of our modelling and study in present paper. The mutual influence between an oscillating system and the mechanism of its excitation, when the later has limited power, gives rise to a number of unusual phenomena in their behaviour [8-11]. The effects of the interaction of an electro-dynamic shaker powered by a vacuum-tube amplifier of limited power, and a linear oscillator which affects the amplitude and frequency of the driving force, are studied in this paper.

1. THE MATHEMATICAL MODEL WITH STRONG INTERACTION

Let us consider an oscillator with damping, mounted on the base of a shaker which undergoes displacements $w(t)$ (Fig. 1). The equation of vibrations of the oscillator of mass m with the vibrational resistance coefficient β_0 has the following form

$$m\ddot{x} + \beta\dot{x} + cx = -m\ddot{w}. \qquad (1)$$

The base of the shaker has a displacement $w(t)$ as a result of the action of the force $H_0 i_0$ [6, 7] applied to the coil L_1, which is rigidly attached to the base. The quantity H_0 is a constant characterizing the electromagnetic field of the vibrator; i_0 is the current of the shaker circuit. The law of motion of the centre of mass of the coil with the base (their mass is m_1) and the oscillating system may be written in the form

$$m_1\ddot{w} + m(\ddot{w} + \ddot{x}) = H_0 i_0 . \qquad (2)$$

The current of the shaker is related to the amplifier current $(i_2 + i_3)$ and the displacement $w(t)$ by the differential relationship [5, 7]

$$(L_0 + L_1)\frac{di_0}{dt} - M\frac{d(i_2 + i_3)}{dt} + H_0\frac{dw}{dt} = 0 \qquad (3)$$

Suppose that the tube operates under conditions when the anodic current equals [5]

$$i_a = a_0 + a_1(e_g + De_a) - \varepsilon a_3(e_g + De_a)^3 , \qquad (4)$$

where e_g is the tube grid voltage; e_a is the anodic voltage; D is the penetration factor of the tube; and ε is a small positive parameter. Applying the method of contour currents, we can write the following Kirchhoff's equations for each branch of the generator current:

$$i_a = i_1 + i_2 + i_3, \quad e_a = E_a - R_a i_1, \quad e_a = -L_k\frac{di_2}{dt} - R_k i_2,$$

$$L_k\frac{di_2}{dt} + R_k i_2 = \frac{1}{C_k}\int i_3 dt, \quad e_c = -E_c + M_k\frac{di_2}{dt}.$$

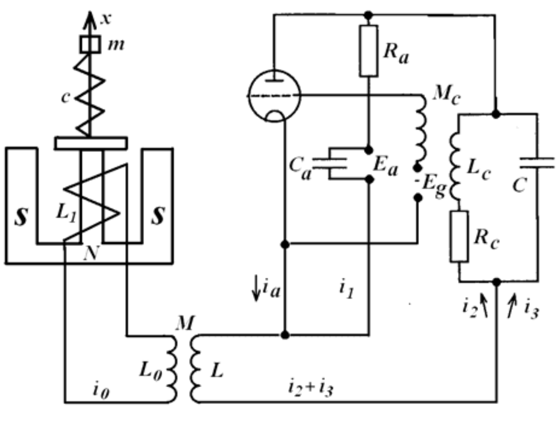


Fig. 1. Scheme of a shaker with an amplifier interacting with an oscillator

After setting up these Kirchhoff's equation for each branch of the amplifier current, let us reduce them to one equation with respect to a new variable

$$u(t) = \int (e_g - E_g)dt$$

($-E_g$ is the constant component of the voltage e_g). We retain only terms of the first order of smallness. Here we assume that

$$\left(L - \left[M^2/\left(L_0 + L_1\right)\right]\right) = \varepsilon\alpha_1, \; D = \varepsilon\alpha_2, \; H_0 = \varepsilon\alpha_3 .$$

Selecting the slope of the tube characteristic in (4), a_1 in accordance with the equation of amplitude balance, we assume it to be equal to

$$a_1 = \frac{R_c R_a C_a + L_c}{R_a(M_c - DL_c)} + \varepsilon a \quad (\varepsilon a > 0).$$

With this value of a_1 we obtain the following nonlinear equation for the function $\mathrm{u}(\mathrm{t})$:

$$\frac{d^2u}{dt^2} + \omega^2 u = \left(-\frac{E_a M_c}{L_c C_c R_a} + a_0 \frac{M_c}{L_c C_c} \right) + \varepsilon a \frac{M_c}{L_c C_c} \frac{du}{dt}$$
$$+\varepsilon a_1 \left(\frac{R_c \omega^2}{R_a L_c} u + \frac{R_c}{R_a{}^2 L_c C_c} \frac{du}{dt} \right) - \varepsilon a_3 \frac{M_c}{L_c C_c} \left(\frac{du}{dt} - E_g \right)^3 + \varepsilon \gamma \dot{x}. \tag{5}$$

Here

$$\omega^2 = \frac{R_a + R_c}{L_c C_c R_a}; \qquad \varepsilon\gamma = \varepsilon\alpha_3 \frac{MM_c m}{L_c C_c R_a (L_0 + L_1)(m_1 + m)}.$$

The tube obtains energy from the energy sources E_a and $-E_g$, which are rectifiers of the supply voltage. We assume the rectifiers to be non-ideal sources of energy [4], since the output voltage E of the rectifier depends on the current i flowing through the load (of the tube oscillator, in this case), according to the external characteristic |5], which is given approximately by $\mathrm{E} = \mathrm{E_{oc}} - \varepsilon \mathrm{ri}$ ($\mathrm{E_{oc}}$ is the open-circuit voltage; $\varepsilon \mathrm{r}$ is a quantity equivalent to the rectifier resistance). Neglecting the grid current, we assume $\mathrm{E_g} = \mathrm{E_{oc}}$. By considering the equality of rectifier output voltage on a shunt of high capacitance $\mathrm{C_a}$ (assuming $\mathrm{C_a} \approx 1/\varepsilon$), we obtain the following relationship for the voltage E_a:

$$E_a = \frac{R_e}{R_e + \varepsilon r} E_{oc} - \varepsilon r_1 u(t) - \varepsilon r_2 \frac{du}{dt} \tag{6}$$

where R_e is the equivalent resistance (the sum of R_a, the tube resistance, and εr); εr_1 and εr_2 are constants determined by the parameters of the tube oscillator and rectifier.

Therefore, E_a is not a constant but depends on the variable function $\mathrm{u}(\mathrm{t})$. This fact clearly must be reflected in the formulation of $\mathrm{u}(\mathrm{t})$. After substituting (6) into (5), the components $(\mathrm{M_c} / \mathrm{R_a L_c C_c})[\varepsilon \mathrm{r_1 u} + \varepsilon \mathrm{r_2}(\mathrm{du}/\mathrm{dt})]$ reflecting the non-ideal character of the energy source of the excitation mechanism, appear on the right side. Terms on the right side of equation (5) may be regarded as ‘internal forces’ and as the effect of interaction with the oscillator. Therefore, we write

$$\frac{d^2u}{dt^2} + \Omega_0^2 u = \varepsilon L\left(u, \frac{du}{dt}\right) - \varepsilon K\left(u, \frac{du}{dt}\right) + \varepsilon\gamma \frac{dx}{dt} \tag{7}$$

Here $\varepsilon L(\mathrm{u}, \mathrm{du}/\mathrm{dt})$ is the sum of the internal forces causing energy influx; $\varepsilon K(\mathrm{u}, \mathrm{du}/\mathrm{dt})$ is the sum of the internal resistance forces; Ω_0 is the frequency of the self-oscillation conditions of the unloaded excitation mechanism; i.e., Ω_0 and amplitude ξ_0 are determined for the function $u = \xi_0 \cos\Omega_0 t$ from the serf-oscillation equation

$$\frac{d^2u}{dt^2} + \Omega_0^2 u = \varepsilon L\left(u, \frac{du}{dt}\right) - \varepsilon K\left(u, \frac{du}{dt}\right) = 0$$

We call the function $\varepsilon L(\mathrm{u},\ \mathrm{du}/\mathrm{dt})$ the static characteristic of the energy source, since under stationary conditions $\varepsilon L(\mathrm{u},\ \mathrm{du}/\mathrm{dt})$ opposes the energy loss $\varepsilon K(\mathrm{u},\ \mathrm{du}/\mathrm{dt})$. These functions have the following form:

$$\varepsilon L(u,\frac{du}{dt}) = \varepsilon\left\{[(a+\frac{r_2}{R_a})\frac{M_c}{L_cC_c}+\alpha_1\frac{R_c}{R_a^2L_cC_c}]\frac{du}{dt}\right\}$$

$$+3a_3\frac{M_c}{L_cC_c}E_g(\frac{du}{dt})^2; \tag{8}$$

$$\varepsilon K(\mathrm{u},\frac{du}{dt}) = \varepsilon a_3\frac{M_c}{L_cC_c}\left\{3E_g^2\frac{du}{dt}+(\frac{du}{dt})^3\right\};$$

and the frequency Ω could be determined from

$$\Omega_0^2 = \omega^2 - \frac{\varepsilon r_1 M_c}{R_aL_cC_c}\mathrm{u} - \frac{\varepsilon\alpha_1 R_c\omega^2}{R_aL_c}\mathrm{u}.$$

We should note that the non-ideal model of the shaker with amplifier (7) has principal difference from the model constructed and used in the papers [8-10], where it has unlimited energy source of variable current. So it is impossible to influence on the frequency what is crucial for stability of the process of interaction [12].

Transforming equations. (1), (2), and (3) and expressing the current (i_2+i_3) by $\mathrm{u}(\mathrm{t})$ enables us to define

$$\frac{d^2x}{dt^2}+\Omega_1{}^2x = \varepsilon\lambda u - \frac{\varepsilon\mu}{\Omega_0}\frac{du}{dt} - \varepsilon\beta\frac{dx}{dt}, \tag{9}$$

where

$$\Omega_1{}^2 = \frac{c(m_1+m)}{mm_1};\quad \varepsilon\lambda = \varepsilon\alpha_3\frac{HMR_c}{m_1M_cR_a(L_0+L_1)};\quad \frac{\varepsilon\mu}{\Omega_0} = \varepsilon\lambda R_aC_c;\quad \varepsilon\beta = \frac{\beta_0(m_1+m)}{mm_1}.$$

Concluding, the system of equations (7) and (9) represents of the coupled shaker-oscillator model with non-ideal amplifier.

2. NUMERICALSIMULATION RESULTS

Introducing the following dimensionless variables:

$$\xi = \frac{u\omega}{E_g},\quad \dot{\xi} = \frac{d\xi}{d\tau} = \zeta,\quad x_1 = \frac{x}{w},\quad \dot{x}_1 = \frac{dx_1}{d\tau},\quad \tau = \Omega_0 t,$$

the system of equations (7) and (9) can be written in the form:

$$\begin{cases} \dot{\xi} = \zeta \\ \dot{\zeta} = -\xi + \gamma_1\zeta + \gamma_2\zeta^2 - \gamma_3\zeta^3 + \gamma_4 p \\ \dot{x}_1 = p \\ \dot{p} = \gamma_5\xi + \gamma_6\zeta - \gamma_0 x_1 - \gamma_7 p. \end{cases} \tag{10}$$

where the coefficients are

$$\gamma_1 = \frac{\varepsilon}{\Omega_0}\left\{(a+\frac{r_2}{R_a})\frac{M_c}{L_c C_c} + \alpha_1 \frac{R_c}{R_a^2 L_c C_c} - 3a_3 E_g^3 \frac{M_c}{L_c C_c}\right\};$$

$$\gamma_2 = 3a_3 \frac{M_c}{L_c C_c} E_g; \gamma_3 = \frac{M_c a_3 \Omega_0}{L_c C_c}; \ \gamma_4 = \frac{\varepsilon\gamma}{\Omega_0}; \ \gamma_5 = \frac{\varepsilon\lambda}{\Omega_0^2}; \ \gamma_6 = -\frac{\varepsilon\mu}{\Omega_0^3}; \ \gamma_7 = \frac{\varepsilon\beta}{\Omega_0};$$

$$\gamma_0 = \frac{{\Omega_1}^2}{\Omega_0^2}.$$

The system (10) is nonlinear, so we may study it numerically. The following values of variables and constants are used in our numerical simulations [7]:

$$E_g = 700V; \ E_a = 2000V; \ a = 6.5\times10^{-5}\ \mathrm{A/V}; \ R_a = 160\Omega; \ R_c = 10\Omega; \ a_3 = 5.184\times X\times10^{-9}\ \mathrm{A/V^3};$$

$$D = 0.015; \ L_c = 0.094H; \ L = 100H; \ M = 1H; \ M_c = 0.275H; \ C_c = 1.0465mF.$$

Using these variables one may obtain the following coefficients for the system (10):

$$\begin{gathered}\gamma_0 = 0.995, \quad \gamma_1 = 0.0535, \quad \gamma_2 = 0.63X, \quad \gamma_3 = 0.21X, \quad \gamma_4 = 0.5 \\ \gamma_5 = -0.0604, \quad \gamma_6 = -0.12, \quad \gamma_7 = 0.01,\end{gathered} \tag{11}$$

X is the bifurcation parameter.

The phase portraits of steady state solutions for the initial conditions $\xi = 0.3$, $\zeta = 0.2$, $x = p = 0.1$ are shown in Fig. 2. The limit cycle graph is shown in Fig. 2 a and corresponds to regular regimes of oscillations [13] with periodically changing variables ξ and ζ. Of course, the variables x and p are also regular and periodic in time. The phase portrait for chaotic regimes of interaction are presented in Fig. 2 b.

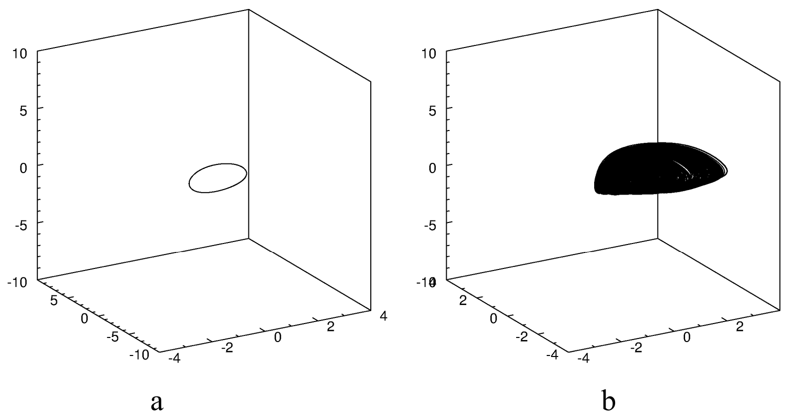


Fig. 2. Graphs of projection of the phase portrait: a – at X=1, b – X=2

The spectrum in Fig. 3 a has discrete peaks. So that, this graph indicates that there is regular regimes in the system at X=1.0. With increasing value of X the transition to chaos occurs. Thus, at X=2.0 chaos is realized in the system, when the spectrum in Fig. 3 b is continuous [13] and the projection of the phase portrait occupies some area in the phase space (Fig.2 b).

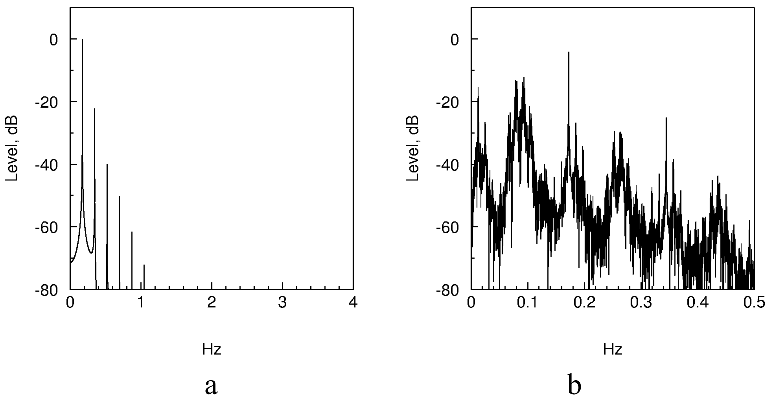


Fig. 3. Graphs of the power spectra: a – at X=1, b – X=2

3. SUPPLIED AND CONSUMED POWERS

Power ratio is easy to obtain from the first integrals of the equations of motion, for which we multiply the first equation (7) on $(\mathrm{du}/\mathrm{dt})$, the second equation by $(\mathrm{dx}/\mathrm{dt})$ and we add both equations. As the result we write

$$\begin{aligned}&\frac{d}{2dt}[(\frac{du}{dt})^2+\Omega_0^2(u)^2]+\frac{d}{2dt}[(\frac{dx}{dt})^2+\Omega_1^2(x)^2]=\\&[\varepsilon L\left(u,\frac{du}{dt}\right)-\varepsilon K\left(u,\frac{du}{dt}\right)+\varepsilon\gamma\frac{dx}{dt}]\frac{du}{dt}\\&+[\varepsilon\lambda u-\frac{\varepsilon\mu}{\Omega_0}\frac{du}{dt}-\varepsilon\beta\frac{dx}{dt}]\frac{dx}{dt}.\end{aligned}\tag{12}$$

After integration in time on the left-hand side of the equation (12) the total energy E of the corresponding conservative system will be presented. The integral on the right-hand side of (12) expresses the sum of the supplied and consumed energies. For steady-state periodic solutions m makes forced oscillations, and the generator generates a periodic current. The energy of the conservative part of the system for steady-state periodic solutions is constant value when integration during the period of the solution. Therefore, the sum of powers of non-conservative part is a periodic function of time but integration of this function in period time is constant. So that the supplied and consumed energies should compensate each other. We may write that the following expression for the powers

$$\begin{aligned}&[\varepsilon L\left(u,\frac{du}{dt}\right)-\varepsilon K\left(u,\frac{du}{dt}\right)+\varepsilon\gamma\frac{dx}{dt}]\frac{du}{dt}\\&+[\varepsilon\lambda u-\frac{\varepsilon\mu}{\Omega_0}\frac{du}{dt}-\varepsilon\beta\frac{dx}{dt}]\frac{dx}{dt}\end{aligned}\tag{13}$$

should be periodic function for the periodic solutions, quasi-periodic function for the quasi-periodic solutions and chaotic for the irregular steady-state regimes. For the two last regimes there is no constant time period integration the expression (13) during which gives the zero value.

Using the dimensionless variables (10) the expression (13) for the total power P can be present in the form

$$\begin{aligned}&(\gamma_1\zeta+\gamma_2\zeta^2-\gamma_3\zeta^3+\gamma_4 p)\zeta+\\&(\gamma_5\xi+\gamma_6\zeta-\gamma_7 p)p=P.\end{aligned}\tag{14}$$

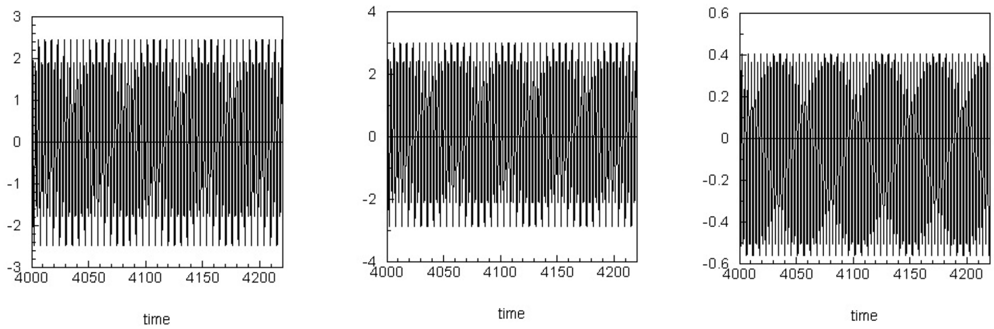
The supplied power P_1 is equal to

$$(\gamma_1\zeta+\gamma_2\zeta^2-\gamma_3\zeta^3)\zeta+(\gamma_5\xi+\gamma_6\zeta)p=P_1\,.\tag{15}$$

The consumed power P_2 could be presented as

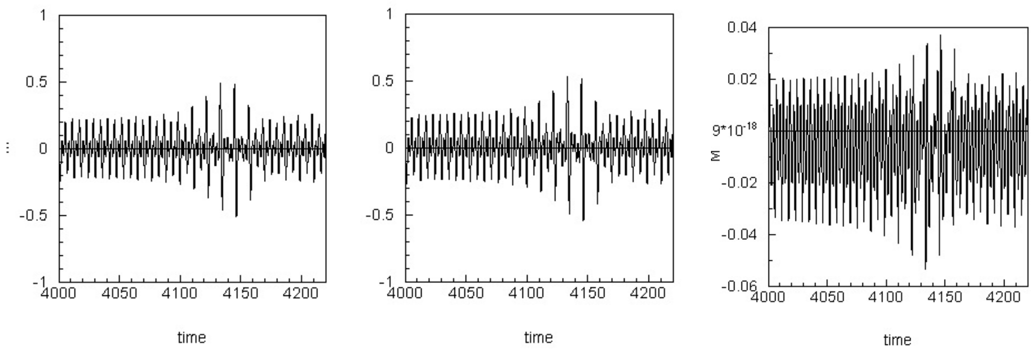
$$(\gamma_4\zeta-\gamma_7 p)p=P_2.\tag{16}$$

In Fig. 4 the powers P, P_1 and P_2 are showed for the coefficients (11) and the same initial conditions as for in Fig. 2 and 3 for the periodic solution at X=0.1. It is clear could be seen that P_2 is oscillating around the negative value (-0.0796) and has amplitudes in order smaller than P_1, which is oscillating around the small positive value (0.0796). If we integrate the power P_2 over its largest period T, then the quantity will be negative as expected (*-0.05T*). The total power P has the mean value equals to zero and a periodic behavior. If we integrate the total power over its longest period, then the integral gives a zero value (the same as the value of integral of left-hand side of equation (12).



a b c

Fig. 4. Graphs of the powers at X=0.1:
a – the total power P, b – the supplied power P_1, c – the consumed power P_2



a b c

Fig. 5. Graphs of the powers at X=3.5:
a – the total power P, b – the supplied power P_1, c – the consumed power P_2

Behavior of powers for chaotic steady-state regimes is much complicated. In Fig. 5 the powers P, P_1 and P_2 are presented for the chaotic solution of the system (10) at X=3.5. For that case P_2 (Fig. 4 c)) is irregularly oscillating around the small negative value (-0.0051) and has amplitudes in order smaller than P_1, which is chaotically oscillating around the small positive value (0.0051). Thus, the total power P has the mean value equals to zero and a chaotic behavior in this case. There is no constant periods for chaotic regimes, so an integral value will have chaotic oscillations around zero value.

4. THE WAVE ENERGY CONVERTER

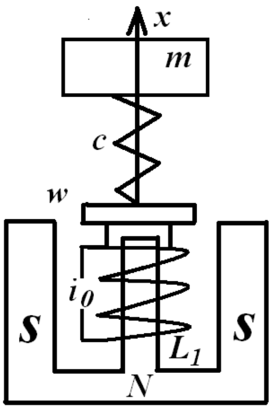


Fig. 6. Schema of wave energy converter

Let us consider the inverse problem: generation of electric current by fluid wave field. We build the most simple a wave energy converter, so called WEC. Let's the base with a coil L_1, and a magnetic field is immersed in fluid with wave motion. Then the motion of the base with a coil L_1, $w(t)$ excites an electric current according Lorenz law. This current is related to the displacement $w(t)$ by the differential relationship [5, 7]

$$(L_0)\frac{di_0}{dt} + H_0\frac{dw}{dt} = 0;$$

The quantity H_0 is a constant characterizing the electromagnetic field; i_0 is the exciting current of the circuit.

The law of motion of the centre of mass of the coil with the base (their mass is m_1) may be written in the form

$$(m+m_1)\ddot{\mathrm{w}} + \mathrm{B}\dot{\mathrm{w}} + \mathrm{Cw} = \mathrm{F_s} + H_0 i_0.$$

Here *m* is the added mass of the fluid wave motion; B is a radiation damping coefficient; C is a constant arising from any restoring force [D.V. Evans and R. Porter, 2012]. And the Ampère force $H_0 i_0$ [6, 7] applied to the coil L_1, which is rigidly attached to the base.

Where $\mathrm{F_s}$ is the excitation force owing to the fluid waves on the device [Evans & Porter]. For simplicity of analysis we assume that the Ampère force $H_0 i_0$ is negligible small as compared with the excitation force $\mathrm{F_s}$. We may omit it.

A power of the considering system could be derived by multiplying the equation (i.2) by $\dot{w}$ and putting the conservative terms in the left hand side of the equation, others in the right hand side. As a result we have

$$\frac{d}{2dt}[(\frac{dw}{dt})^2 + \frac{C}{m_1 + m}(\mathrm{w})^2] = (\mathrm{F}_s - B\frac{dw}{dt})\frac{dw}{dt}$$

We may rewrite the right hand side of the equation presenting the supplied and consumed powers as

$$\mathrm{W} = (\mathrm{F}_s - B\frac{dw}{dt})\frac{dw}{dt} = \frac{F_s^2}{4B} - B\left[\frac{dw}{dt} - \frac{F_s}{2B}\right]^2;$$

Therefore the maximum power is

$$\mathrm{W}_{\max} = \frac{F_s^2}{4B}$$

If F_s is the excitation force owing to the fluid waves on the device then F_s and *w* are oscillating function in time. Let us assume they are periodical in time. The mean power W^*, generated by the fluid waves on the device is the time averaged over a period time.

So that

$$\mathrm{W}^* = \frac{1}{2}(\mathrm{F}^*{}_s - B\dot{w}^*)\dot{w}^*$$

Where $\mathrm{F}^*{}_s$ is the amplitude of F_s and $\dot{w}^*$ is that amplitude of w which is in phased with F_s. Now

$$\mathrm{W}^*{}_{\max} = \frac{F^{*2}{}_s}{8B}$$

CONCLUSIONS

The coupled shaker-oscillator model, which takes into account both direct and reverse influence of subsystems is worked out. The methods of modern theory of the dynamical systems are used to study laws of the steady-state regimes of the complex model with strong interaction. The chaotic regimes were found out. The dynamics of the oscillator system is in good correspondence with experimental information of a limited power shaker behavior [9, 10, 11]. Found irregularities of phase trajectories of the complex model depend on intensity of the amplifier tube. The total power, the supplied power and the consumed power are defined and calculated for the periodic and chaotic steady-state regimes. It was shown that the total power is oscillating along zero mean value, when the consumed around negative and supplied around small positive value.

In the inverse problem: classical linearized water wave theory is used to develop expressions for the power absorption for a particular power take-off mechanism and the maximum theoretical power absorption. The advantage of our approach is that we find the powers as a function of time, and not just the averaged quantities. And we can calculate them for any regular or chaotic regimes.

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