ON THE HIGH FREQUENCY VIBRATIONS OF THE PIEZOCERAMIC DISK

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The resonant frequencies of Pb(ZrTi)O3 disk were studied. In the framework of elastic isotopic body the experimental frequencies were characterized. The theoretical solution of finite disk vibrations constructed of the displacements in the series of trigonometric and Bessel functions of the thickness and the radial directions respectively. Theoretical frequencies were found with necessary accuracy. In the low frequency range the edge resonance was predicted with high accuracy. The specific types of vibrations in high frequency range were described theoretically and confirmed experimentally. General good agreement in resonant frequencies between the theoretical and the experimental results was found.

INTRODUCTION

This work is devoted to analysis of the frequency spectrum of the piezoceramic disk made from Pb(ZrTi)O3 material. The investigation is based on the theoretical model of steady vibrations of an isotropic finite cylinder. The solution of the problem is the result of an expansion of the displacements in the series of trigonometric and Bessel functions of the thickness and the radial coordinates respectively. The method of solution according to [1] will be called “method of superposition”. When the analytical solution was obtained the properties of the experimental pattern were found. Then the theoretical frequencies were compared with the experimental ones. Good agreement even for so called edge-modes [2] was found, although Mindlin’s second order theory predicts edge resonances but they lower than experimental ones by value about 14% for BaTiO3 disks [4]. Edge modes were discovered by E.A.G. Shaw [2], distinct the motion localized at the circular boundary of a disk and decreased rapidly toward the center.

The attention is also paid to high frequency vibrations, where frequencies higher than the frequency minimum of the second branch in Fig.1 with its associated zero group velocity at a nonzero wave number and phase and group velocities of opposite signs at smaller wave numbers. Possibility of very accurate predicting the piezoceramic disk frequencies up to the frequency of thickness-shear mode was concluded. In the latter the displacements are parallel to the middle plane of the infinite plate. In this work Poisson’s ratio ν grater then 1/3 and the frequency of the thickness-extensional mode is higher than the frequency of symmetric thickness-shear mode. The thickness-extensional mode existing in an infinite plate with displacements is normal to the middle plain of the plate and the middle plain is the nodal plain. The frequency of this mode depends on Poisson’s ratio ν . The results of proposed investigation may be summarized as following: it is found that in the frequency range of backward wave specific types of modes called B modes and A modes [5] exist for the material with Poisson’s ratio grater then 1/3.

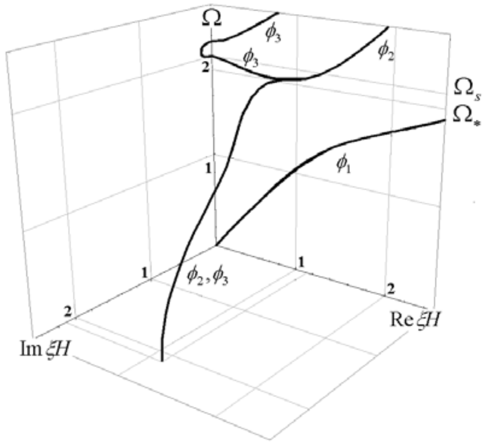


Figure 1

THEORETICAL ANALYSIS

The solution of Rayleigh’s equation for infinite elastic plate of thickness 2H presented in Fig.1 is necessary for further analysis. The finite number of real propagate constant $\mathrm{Re}\ \xi$ or imaginary $\mathrm{Im}\ \xi$ and infinite number of complex conjugate propagate constants $\xi, \bar{\xi}$ the spectrum has for the fixed normalized frequency $\Omega = fH/c_2$ (f – frequency in Hz, c_2 – shear wave velocity, H – semi-thickness of plate).

To consider the problem for vibrations of a finite cylinder let us put the cylindrical coordinate system $Or\theta z$ in the center of a cylinder. The even particle in a cylinder should satisfy axis symmetric Lame equations:

$$\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} + \frac{\partial^2 u_r}{\partial z^2} - \frac{u_r}{r^2} + \frac{\nu}{1-2\nu}\frac{\partial \Theta}{\partial r} + \gamma_2^2 u_r = 0,$$
$$\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r}\frac{\partial u_z}{\partial r} + \frac{\partial^2 u_z}{\partial z^2} + \frac{\nu}{1-2\nu}\frac{\partial \Theta}{\partial z} + \gamma_2^2 u_z = 0, \qquad (1)$$

where ω – angular frequency; $\gamma_1 = \omega / c_1$, $\gamma_2 = \omega / c_2$ – normalized frequencies; $c_1 = c_2\sqrt{2(1+k)}$ – dilatational wave velocity; ν – Poisson’s ratio, $k = \nu/(1-2\nu)$; $c_2 = \sqrt{G/\rho}$ – shear wave velocity; $\Theta = \frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_r}{\partial z}$ – volume dilatation, G – shear modulus and ρ – material density, harmonic factor $e^{i\omega t}$ will be omitted here and in what follows.

On the cylinder boundaries the stresses have to satisfy the following conditions:

$$\sigma_r(a,z) = 0, \qquad \tau_{rz}(a,z) = 0, \qquad -H \le z \le H, \qquad (2)$$

$$\sigma_z(r,\pm H) = g(r), \qquad \tau_{rz}(r,\pm H) = 0, \qquad 0 \le r \le a.$$

Here $\sigma_r(a,-z) \equiv \sigma_r(a,z)$, means only symmetric vibrations with respect to the plane z=0. The stresses and the displacements are related by means of Hook's law and Caushy relations.

The solution of the problem (1)-(2) is taken as the series of the complete and the orthogonal trigonometric and Bessel functions

$$u_r = x_0 \frac{J_1(\gamma_1 r)}{\gamma_1} + a\sum_{n=1}^{\infty} x_n (-1)^n \left[\frac{I_0(q_2 r)}{I_1(q_2 a)} - \frac{k_n^2 + q_2^2}{2k_n^2}\frac{I_0(q_1 r)}{I_1(q_1 a)}\right]\cos(k_n z) + H\sum_{j=1}^{\infty} y_j \left[\frac{\lambda_j^2 + p_2^2}{2\lambda_j p_1}\frac{\cosh(p_1 z)}{\sinh(p_1 H)} - \frac{p_2}{\lambda_j}\frac{\cosh(p_2 z)}{\sinh(p_2 H)}\right]\frac{J_1(\lambda_j r)}{J_0(\lambda_j a)};$$

(3)

$$u_z = y_0 \frac{\sin(\gamma_1 z)}{\gamma_1} + a\sum_{n=1}^{\infty} x_n (-1)^n \left[\frac{k_n^2 + q_2^2}{2k_n q_1}\frac{I_0(q_1 r)}{I_1(q_1 a)} - \frac{q_2}{k_n}\frac{I_0(q_2 r)}{I_1(q_2 a)}\right]\sin(k_n z) + H\sum_{j=1}^{\infty} y_j \left[\frac{\sinh(p_2 z)}{\sinh(p_2 H)} - \frac{\lambda_j^2 + p_2^2}{2\lambda_j^2}\frac{\sinh(p_1 z)}{\sinh(p_1 H)}\right]\frac{J_0(\lambda_j r)}{J_0(\lambda_j a)}.$$

Here $q_i = k_n^2 - \gamma_i^2$, $p_i = \lambda_j^2 - \gamma_i^2$, when $i = 1,2$; and $k_n = \pi n / H$, $J_1(\lambda_j a) = 0$ for $j, n = 1,2,...$.

The coefficients of the series (3) should be found from the infinite system of linear algebraic equations which are the result of fulfilling the boundary conditions (2). The important conclusion in analysis of the infinite system was made by V.T. Grinchenko [1], he proved that on frequencies which are not equal to the resonant ones the unknown coefficients have the following asymptotic

$$\lim_{n\to\infty} x_n = -\lim_{j\to\infty} y_j = a_0, \qquad (4)$$

where $a_0 = const$. The rule (4) let us pass from the infinite system to the finite thus to find values of the resonant frequencies with necessary accuracy.

EXPERIMENTAL WORK

The disk from Pb(ZrTi)O3 piezoceramic material was studied on the resonant vibrations. The plane surfaces of the disk were fully covered by split silver electrodes therefore the cylinder had the polarization along the thickness. The piezoelectric cylinder had the diameter of 70 mm, the thickness equal to 8 mm and the density equal to 6821 kg/m^3. The frequencies spectrum was measured from 10 kHz up to 300 kHz. The resonant frequencies were measured at the maximum values of voltage *U*. It should be mentioned that only symmetric frequencies relatively to angular coordinate and central plane were electrically excited that means that the specimen had been machined precisely.

DISCUSSION

The information concerning the elastic constants of the disk was obtained. Although piezoceamic is transversely isotropic material the assumption of its isotropic doesn't lead to serious discrepancies between the theoretical and the experimental results. Two elastic constants are needed if a disk is assumed to be isotropic. Poisson's ratio and the shear wave velocity are more convenient. Poisson's ratio for Pb(ZrTi)O3 is 0.361 and the shear wave velocity is 1807.5 m/s. The elastic characteristics were calculated by interpolation of the theoretical and the experimental results as described in [6]. The frequency spectrum in Fig. 2 for normalized frequency Ω versus dimensionless radius *R=a/H* is presented. The frequencies 1–4 are out of our interest and are not shown in Fig. 2. The frequencies 1–5 are essentially radially dilatational in character until Ω approaches Ω_e =1.44. It can be shown by approximately linear relationship between *R* and Ω and by the normal surface displacements patterns which are single Bessel functions $J_0(\lambda_j r)$ at *j*=1,2,…,5. When each spectral curve approaches the region at the plateau corresponding Ω_e frequency Ω becomes essentially independent from *R* and the displacements u_z at the plane surface *z*=*H* are characterized with the exponential decay toward the center of a cylinder. The resonant frequencies 7–8 are also radial.

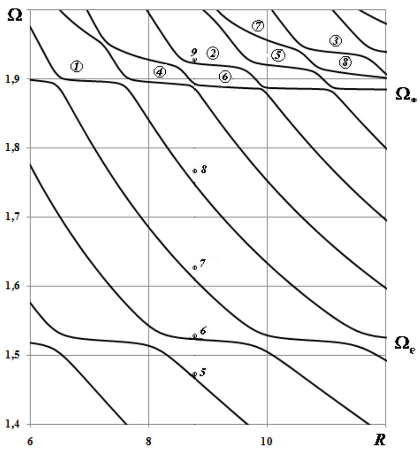
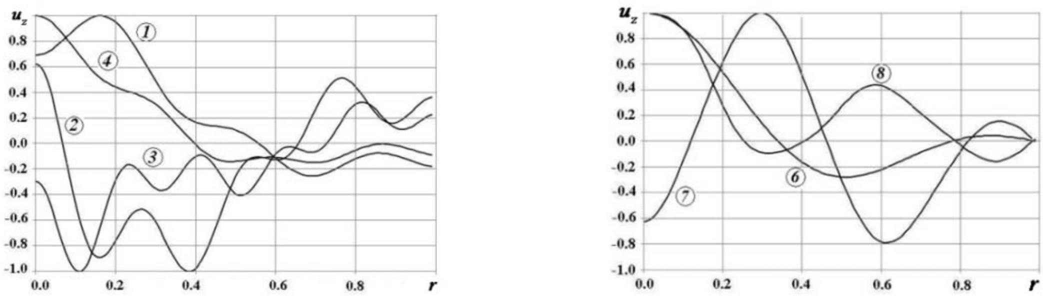


Figure 2

Other interesting resonances appear in the frequency range $\Omega_* < \Omega < \Omega_s$. The frequency Ω_* =1.88 is the frequency minimum of the second branch ϕ_2 in Fig. 1 and the group velocity is equal to zero with nonzero propagation constant. That phenomenon produces the typical terrace like structure of the high frequency spectrum of disks. It begins to develop at the frequency with asymptotic value Ω_* when $R \to \infty$. The aforementioned resonances construct “stage-structure” in that frequency range, which was described in [5] for the case when ν <1/3. A classification for ν =0.361 can be made only on the basis of analysis of the vibration form u_z on the individual segments of the corresponding spectral curve (see Fig. 3). It was concluded that stage-structure is saved for ν >1/3 and consists from B **aaa**– **aaa** in Fig. 3 (a) and A modes **aaa**– **a** in Fig 3 (b). The number of B modes increase with increasing dimensional radius *R*, they develop up to the frequency with asymptotic value Ω_s when $R \to \infty$. In accordance with [5] the number of nodal circles in displacement u_z corresponds the order of B and A modes**aaaa**–**aaa** B_2 mode and **aaa**,**aaa** construct B_1 mode. The segment of spectral curves **aaa** is A_2 mode and **aaa**,**aaa** are A_3 mode.

Let us pass to comparison of the theoretical and the experimental results. The experimental resonant frequencies are presented as the dots with corresponding numbers 5–9 in Fig. 2. One could see good general agreement between considering results. The resonant frequencies 1–5 have discrepancy of about 1 %, but frequencies 7, 8 give about 4 %. The proposed method gives good agreement for the experimental value of the edge resonance and discrepancy of less than 1% at while the second order theory gives 16% [3]. At the same time improved second order theory by P.C.Y. Lee is good at predicting the modes of edge resonances [5]. The resonant frequency 9 was found theoretically with discrepancy of about 0.8 % and it belongs to B_2 mode. The frequency of B_1 and A_2 modes was not found experimentally and the reasons for its not having been detected require further study.



(a) (b)

Figure 3

CONCLISIONS

The experimental edge-mode was predicted theoretically with discrepancy of about 1%. Existence of B and A modes for $\nu > 1/3$ in a finite disk was proved. The existence of B mode was confirmed experimentally for Pb(ZrTi)O3 disk. When A mode was not excited in the experiment, the other resonant frequencies were predicted theoretically with high accuracy.

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