STABILIZATION OF THE FLUID FLOWS IN THE MULTILAYERED TUBES FROM VISCOELASTIC MATERIALS

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Stability of the incompressible fluid flow through a thick-walled multilayered anisotropic viscoelastic tube is studied. Both steady laminar and turbulent basic flows are considered. The temporal and spatial eigenvalues of the system are found. Influence of the material parameters of the layers and the Reynolds number on the spatial and temporal amplification rate of the most unstable mode is investigated. It is shown that the absolute instability of the system can be converted into a convective instability, and in some cases the system can be stabilized by an appropriate choice of the rheological parameters. The results can be applied to the blood flow in vessels and fluid flows in compliant tubes of biomedical and technical devices, for damping the undesirable solid vibrations and noise shielding.

INTRODUCTION

When fluid moves over a deformable surface or through the tubes and channels of different technical devices, in the heat and mass exchangers, mixture separation and purification systems, polymer processing and oil pipelines different phenomena caused by fluid-structure interaction can be observed. Flow and pressure limitation phenomena, self-excited oscillations, vibrations of the surfaces and noise generation are often observed in the biomedical ad technical systems. The flow inside the tubes often creates turbulent boundary layers, which can lead to significant loss in efficiency. The transition from laminar to turbulent flow is responsible for the high drag experienced by aircrafts, ships and underwater vehicles. Reducing turbulent sound generation, which is the main noise source inside the vehicle, is of a great interest for industry.

An avalanche of the experimental and theoretical investigations of the flow stability past the compliant surface had been promoted by observations of the swimming dolphins. Kramer (1957) was the first who suggested a dolphin's body experiences low drag due to special structure of its compliant skin, which allows delaying transition to turbulence and maintaining the laminar flow. The idea was used for the nature inspired solution and it was shown the compliant walls with certain rheological parameters can be used for drag reduction, delay of the laminar-to-turbulence transition, noise suppression, prevention of the boundary-layer separation, sound absorption and vibration damping due to energy transfer at the fluid-solid interfaces [1].

Flows in compliant tubes abound in living organisms. Blood flow through arteries and veins; flow through stenosed, dilated and stented blood vessels; air flow in nasal cavities and airways, urine flow through urethra exhibits phenomena produced by the fluid-solid interaction and flow instability. The results can be both favourable and unfavourable for the organisms. Air flow through collapsible upper airways leads to the sound production and flow-limitation phenomena in snorers. Air flow instability in glottis and larynx is the main mechanisms of the speech generation. Oscillations of the tube and periodic flow limitation provides blood delivery to the brain in the long-necked animals.

Stability of the steady flow of the viscous incompressible fluid in the three-layered anisotropic tube has been studied in [2,3] in application to the blood flow in the arteries and veins. It was shown that variations of the elastic modules and viscosities of one of the layer or two layers simultaneously exert a great influence on the stable and unstable modes. The effect of the shear modulus of each layer on the amplification rate has been found to be different and the shear modules of the inner and middle layers produced opposite effects on the system stability. The shear modulus of the middle layer significantly influences the amplitude of the group velocity of the most unstable mode.

Effect of viscosity of the wall layers on the flow stability has been studied for both the no stress [4] and no displacement [4,5] boundary conditions at the outer surface of the tube. It was shown that any increase in the viscosity of the inner layer leads to an increase in the amplification rate of the most unstable mode because the more viscous inner layer enhances the fluid-solid interaction at the interface. The viscosity of the middle layer exerts a stabilizing effect on the amplification rate of the most unstable mode. Both the cases are proper to the blood vessels. The no displacement conditions correspond to rigid attachment of the vessel wall to the outer tissues (deep intraorgan blood vessels) while the no stress conditions correspond to the free surface (superficial extraorgan blood vessels). In the technical devices both the conditions are also can be valid depending on the design of the rigid and compliant surfaces. Here the problem is studied in application to the turbulent flow regime in the multilayered tube and the comparative analysis of the results for the steady and turbulent flows is given.

1. PROBLEM FORMULATION

The flow of the incompressible viscous fluid through the three-layered viscoelastic tube with the inner radius R, thickness h and length L is considered. The wall is composed of three non-isotropic layers with thicknesses h_1, h_2, h_3, where $h_1 + h_2 + h_3 = h$ (fig.1).

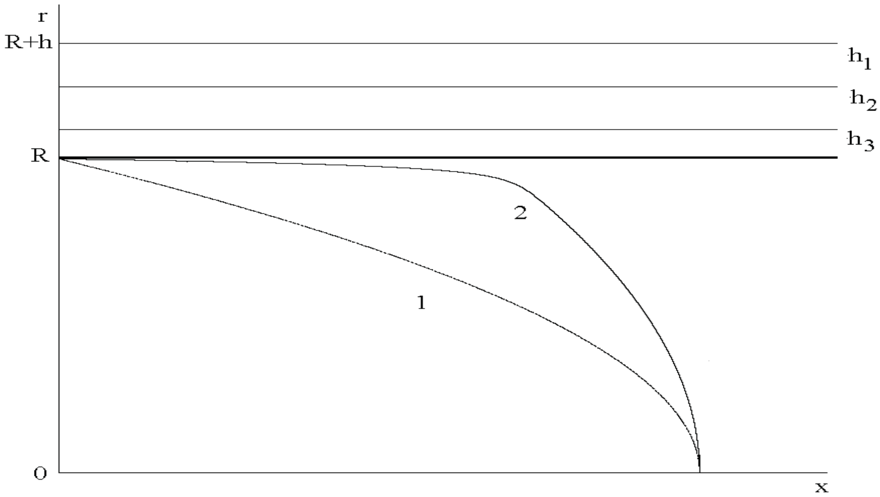


Fig.1. Steady (1) and turbulent (2) basic velocity profiles.

The mass and momentum conservation equations are the incompressible Navier-Stokes equations for the fluid and the classic viscoelastic body equations for the wall:

$$div(\vec{v}) = 0\,,\ \rho_f \frac{d\vec{v}}{dt} = -\nabla p + \mu_f \Delta\vec{v} \qquad (1)$$

$$div(\vec{u}^{(j)}) = 0\,,\ \rho_s^{(j)} \frac{\partial^2 \vec{u}^{(j)}}{\partial t^2} = -\nabla p_s^{(j)} + div(\hat{\sigma}^{(j)}) \qquad (2)$$

where $\vec{v}$, p, ρ_f and μ_f are the fluid velocity, hydrostatic pressure, density and viscosity; $\vec{u}^{(j)}$, $p_s^{(j)}$, $\hat{\sigma}^{(j)}$ and $\rho_s^{(j)}$ are the displacement, hydrostatic pressure, stress tensor and density of the j-th layer, $j = 1{,}2{,}3$.

For the viscoelastic layers the Kelvin-Voight model of the viscoelastic body is used and the constitutive equations are

$$\left(\tau_s^{(j)} \frac{D}{Dt} + 1\right)\sigma_i^{(j)} = A_{ik}^{(j)}\varepsilon_k^{(j)} + \mu_s^{(j)} \frac{D}{Dt}\varepsilon_i^{(j)} \tag{3}$$

where $\sigma_i^T = \left(\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}, \sigma_{\theta z}, \sigma_{rz}, \sigma_{r\theta}\right)$, $\varepsilon_i^T = \left(\varepsilon_{rr}, \varepsilon_{\theta\theta}, \varepsilon_{zz}, \varepsilon_{\theta z}, \varepsilon_{rz}, \varepsilon_{r\theta}\right)$, $\sigma_{ik}^{(j)}$ and $\varepsilon_{ik}^{(j)}$ are stress and strain tensors for the j-th layer, $A_{ik}^{(j)}$ is the matrix of elastic modules, $\mu_s^{(j)}$ is the viscosity of the j-th layer.

The matrices $A_{ik}^{(j)}$ are considered in the form proper to the orthotropic materials:

$$\left(A_{ik}^{(j)}\right)^{-1} = \begin{pmatrix} (E_1^{(j)})^{-1} & -\nu_{21}^{(j)}(E_2^{(j)})^{-1} & -\nu_{31}^{(j)}(E_3^{(j)})^{-1} & 0 & 0 & 0 \\ -\nu_{12}^{(j)}(E_1^{(j)})^{-1} & (E_2^{(j)})^{-1} & -\nu_{32}^{(j)}(E_3^{(j)})^{-1} & 0 & 0 & 0 \\ -\nu_{13}^{(j)}(E_1^{(j)})^{-1} & -\nu_{23}^{(j)}(E_2^{(j)})^{-1} & (E_3^{(j)})^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & (G_1^{(j)})^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & (G_2^{(j)})^{-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & (G_3^{(j)})^{-1} \end{pmatrix}$$

where $E_i^{(j)}$, $G_i^{(j)}$, $\nu_{ik}^{(j)}$ are Young and shear modules and Poisson ratios for the j-th layer.

The boundary conditions are continuity conditions for the velocity and normal and tangential components of the stress tensor at the fluid-solid interface; the continuity conditions for the stress components and displacements at the interfaces between the layers; the no displacement condition at the tethered outer surface of the tube or the no stress condition for the unloaded outer surface which is allowed to move:

$$r = R: \quad \vec{v} = \partial\vec{u}^{(1)}/\partial t, \quad \hat{\sigma}_n^{(1)} = \hat{\sigma}_n \tag{4}$$

$$r = R + h_1: \quad \partial\vec{u}^{(1)}/\partial t = \partial\vec{u}^{(2)}/\partial t, \quad \hat{\sigma}_n^{(1)} = \hat{\sigma}_n^{(2)} \tag{5}$$

$$r = R + h_1 + h_2: \quad \partial\vec{u}^{(2)}/\partial t = \partial\vec{u}^{(3)}/\partial t, \quad \hat{\sigma}_n^{(2)} = \hat{\sigma}_n^{(3)} \tag{6}$$

$$r = R + h: \quad \vec{u}^{(3)} = 0 \quad \text{or} \quad \hat{\sigma}_n^{(3)} = 0 \tag{7}$$

where n denotes the normal component, $\hat{\sigma}$ is the stress tensor for the fluid.

The problem (1)-(7) can be generalized to an arbitrary number of the viscoelastic layers. For the tree-layered isotropic material ($E_{1-3}^{(j)} = E^{(j)}$, $G_{1-3}^{(j)} = G^{(j)}$, $\nu_{ik}^{(j)} = \nu^{(j)}$) and for the transversely isotropic material when the plane of isotropy is the θx -plane that correspond to the properties of the blood vessel wall the problem (1)-(7) has been solved in [2,4]. The layers have different parameters $E^{(j)}$, $G^{(j)}$ and $\nu^{(j)}$, so the layers were isotropic, but the tube possessed anisotropy in the radial direction. A possibility to stabilize the system by a two-layered viscoelastic coating has been studied by Kranzt (1971) and was not successful [5].

Solution of the problem (1)-(7) is considered in the form of a normal mode:

$$\begin{aligned} \{\bar{v}, p\} &= \{\bar{v}^{b}, p^{b}\} + \{\bar{v}^{*}, p^{*}\} exp(st + in\theta + ikx) \\ \{\bar{u}^{(j)}, p_s^{(j)}\} &= \{\bar{u}^{b(j)}, p_s^{b(j)}\} + \{\bar{u}^{*(j)}, p^{*(j)}\} exp(st + in\theta + ikx) \end{aligned} \quad (8)$$

where $\bar{v}^{*}$, $\bar{u}^{(j)*}$, p^{*}, $p_s^{*(j)}$ are the amplitudes of the corresponding disturbances, $k = k_r + ik_i$, $s = s_r + is_i$, s_i is the wave frequency, k_r is the wave number, s_r and k_i are spatial and temporal amplification rates. The steady part $\{\bar{v}^{b}, p^{b}, \bar{u}^{b}, p_s^{*(j)}\}$ of (8) is identified with Poiseuille flow and the turbulence flow for the two cases (fig.1) and the measured basic turbulence flow was taken [6]. The isotropic, transversely isotropic and orthotropic materials for the wall layers have been considered. Both the anisotropic and isotropic walls (all the layers are made form the same isotropic materials) and both nonuniform and uniform walls (all the layers are made form the same anisotropic materials) of the tube are considered.

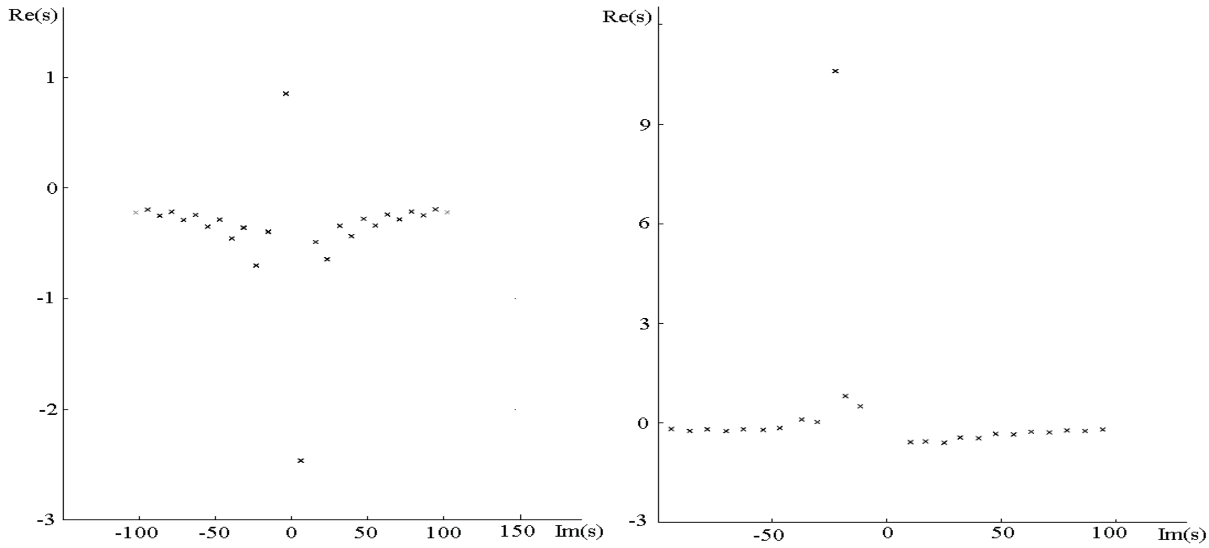
2. NUMERICAL METHOD

A numerical procedure has been developed to solve the coupled fluid (1)-(2) and solid (3)-(4) equations, allowing the computation of s for a given k (temporal eigenvalues) and k for a given s (spatial eigenvalues). The dispersion equation D(s,k) is thus established; the numerical method used to compute the dispersion relation D is presented in [7] and an improved version is explained in [8], the procedure is not repeated here for brevity. The numerical procedure allows the computation of the roots of that dispersion relation as well as its double roots. The procedure includes a fourth-order Runge-Kutta method to solve the differential equations in the fluid and in the solid medium. Matching the boundaries conditions at the fluid/solid interface leads to the dispersion equation; then an iterative technique using the steepest decent method is developed to find the double roots of the dispersion equation.

3. RESULTS AND DISCUSSIONS

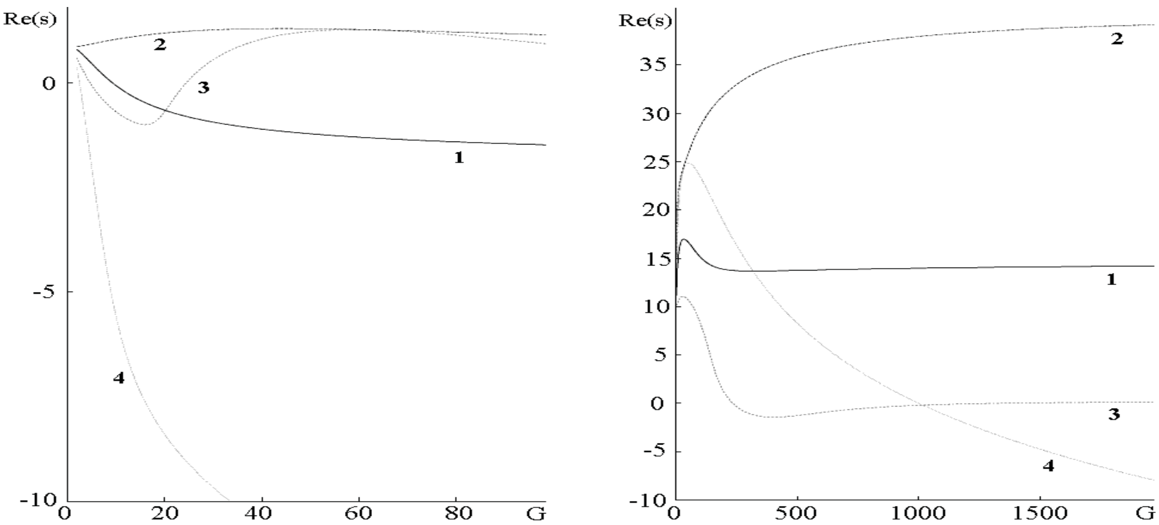
The temporal eigenvalues of the system in the complex (s_r, s_i) - plane are shown for the Poiseuille and turbulent flows in fig.2a and fig.2b accordingly. The modes located near the real axis of the plane are solid based and those located near the imaginary axis are fluid based. The distribution density of the eigenvalues along the real and imaginary axis depends on the elastic and fluid inertia forces determined by the values Γ and Re. For the cylindrical water column with free boundaries the fluid based modes only remain in the (s_r, s_i)-plane, whereas for the empty viscoelastic shell the fluid based modes disappear and we can find the solid based ones only. The modes involved efficiently in the fluid-solid interaction are placed near the origin of the complex plane where the unique unstable mode with $s_r > 0$ is located. The parameters used for the numerical computations are $k = 2.5, n = 0, Re = 10, \Gamma = 10, \rho_s^{*} / \rho_f = 1$, $h / R = 0.4$, $h_1 / R = 0.08, h_2 / R = 0.14, h_3 / R = 0.18$, $\nu^{(j)} = 1$, $G^{(j)} / G^{*} = 1$.

As it can be seen, for the Poiseuille flow at least one unstable mode can be found. As it was proved in our previous papers [2-5] the observed modes are absolute unstable and they can be converted into the convection unstable modes or can be stabilized by an appropriate choice of the Young's modules, shear modules and viscosities of the middle and outer layers. For the turbulence flow we have a wide range of the unstable modes and at least one or two of them are much more unstable than the unstable modes for the Poiseuille flow.



a **b**

Fig.2 Temporal eigenvalues for the Poiseuille (**a**) and turbulent (**b**) flows for the material parameters mentioned in the text.



a **b**

Fig.2. Amplification rate and frequency of the most unstable mode versus the non-dimensional shear rate for the Poiseuille (**a**) and turbulence (**b**) basic flows and different sets of the non-dimensional shear rates of the wall layers: $G^{(1)} = G^{(2)} = G, G^{(3)} = 1$ (curve 1), $G^{(1)} = G^{(3)} = G, G^{(2)} = 1$ (curve 2), $G^{(2)} = G^{(3)} = G, G^{(1)} = 1$ (curve 3), $G^{(1)} = G^{(2)} = G^{(3)} = G$ (curve 4).

The system can be stabilized by a proper variation of the shear modules of one of the wall layers, as it is shown in fig.3. As it was found, the steady flow in the distensible tube can be stabilized (Re(s) < 0) by the rigid inner layer (curve 1 in fig.3a) and by certain increase in the rigidity of the outer layer (fig.3 in fig.3a) while the rigidity of the middle layer exerts an destabilizing influence on the flow. Obviously, rigidity of all the layers simultaneously stabilizes the flow (curve 4 in fig.3a). As it can be seen in fig.3b, the turbulent flow can be stabilized by the complete rigid wall for the high enough Re numbers (curve 4 in fig.3b) or by certain range of the Young’s modules of the outer layer. It means, the inner viscoelastic bilayer can absorb some energy and stabilize the flow in the tube and suppress the wall oscillations and noise.

CONCLUSIONS

As it was shown by our numerical computations, stability of the flow in the three-layered viscoelastic tubes can be achieved for different basic flows. For the Poiseuille fllow an increase in the shear modulus of the inner and middle layers decreases the temporal amplification rate and stabilizes the system whereas some increase in rigidity of the outer layer eliminates the temporal instability of the steady viscous flow in the compliant tube. The comparative analysis of the system stability at the no displacement and no stress boundary conditions at the outer surface of the duct revealed that stabilization of the system can be provided by increasing the rigidity of the inner layer in the both cases. For the transversely isotropic material the temporal stability can be achieved by increasing the shear modulus in the plane of isotropy of any of the layer at the no stress boundary conditions and by increasing the viscosity of the second layer at the no displacement conditions. A turbulent flow in the distensible tube is much more unstable, but for some geometry the flow can also be stabilized.

The obtained results on the flow stabilization by a proper choice of the rheological parameters of the wall can be used for the flow stabilization in the ducts and tubes of technical and biomedical devices. Concerning the blood vessels stabilization can be naturally increased or decreased depending on whether it is favourable or unfavourable for the physiological function.

LITERATURE

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