ISSN 1028-7507 Акустичний вiсник. 2000. Том 3, N 2. С. 3–9

517.52

HYPERBOLIC EQUATIONS IN WAVELET BASES

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Received 16.06.2000

In this paper we discuss the problem of representing hyperbolic differential operators using orthonormal wavelet bases. A numerical algorithm is shortly outlined and a numerical experiment in the linear magnetohydrodynamics, as a test problem, is eventually outlined.

В статті обговорено проблему представлення гіперболічних диференційних операторів з використанням ортонормованих вейвлет-базисів. Коротко описано розрахунковий алгоритм, а також чисельний експеримент з області лінійної магнітогідродинаміки, який виступає в ролі тестової задачі.

В статье обсуждена проблема представления гиперболических дифференциальных операторов с использованием ортонормированных вейвлет-базисов. Коротко описан расчетный алгоритм, а также численный эксперимент из области линейной магнитогидродинамики, выступающий в роли тестовой задачи.

INTRODUCTION

Wavelet theory [1, 2] has been quite recently applied to the numerical modelling of differential equations [3–6]. It founds wide application in modern acoustics and hydrodynamics when desctribing the processes going with different time-frequency scales.

The main feature of this method is providing an efficient approximation for functions in the wavelet bases of a suitable functional space. Furthermore, from the point of view of the numerical computation, fast algorithms for the wavelet transform are available [1, 7], and these algorithms are proven to be much faster than the well known (and celebrated) fast Fourier transform (see e.g. [7, 8]).

A localized analysis [1, 2] seems to be the most convenient approach for studing phoenomena, in particular nonlinear. In fact, impulse functions and distributions, performing singularities, are well represented by their spectral decomposition in a wavelet bases. Wavelets are very well localized functions [1, 2], and, since they are zero nearly everywhere, they can be easily treated in numerical applications. The numerical approach to a differential equation is based on the representation of the unknown function, with its corresponding derivatives in a wavelet bases, and like in the collocation (or pointwise) method, the value of the unknown function will be a linear combination of known values at sampling locations. We will choose the Haar family of wavelets as bases, and the coefficients will be computed by the fast Haar transform (see e.g. [2, 4, 7, 8]). The problem of representing the derivatives of the unknown function will be solved smoothing the Haar wavelets with suitable order splines.

As application of this method we will approach the fundamental equations of magnetohydrodynamics (section 1) which are based on a hyperbolic system of differential equations (in general non symmetric, see section 2). Theorems for the existence and uniqueness solution of hyperbolic system of magnetohydrodynamics in suitable spaces (section 3), are known (see e.g. [9] and therein quoted references), also in some special case (see [9] for dissipative boundary conditions), but exact solutions are always hardly recovered. In this paper it is shown the construction of the Haar wavelet(section 4) solution of this hyperbolic system. Wavelets are bases in the so-called adaptive spaces (section 5); the collocation method referred to these, with a smoothing process of the Haar wavelets, will be discussed in section 6. The explicit numerical method is eventually sketched in section 7.

1. MAGNETOHYDRODYNAMICS

Let Ω be a domain of R^3, $x=(x_1, x_2, x_3)$ an arbitrary point and I a finite interval of the time variable t,

$$I \overset{\text{def}}{=} \{t : 0 < t < T,\ T < \infty\},$$

$$Q \overset{\text{def}}{=} \Omega \times I.$$

The electric field, the magnetic field and the speed of electrons are the vectors $\mathbf{H}, \mathbf{E}, \mathbf{v}$ respectively; p is the scalar pressure of the electrons. The equations of magnetohydrodynamics are summarized by the system (see e. g. [9])

$$\begin{cases} \mu_o \dfrac{\partial \mathbf{H}}{\partial t} = -\nabla \times \mathbf{E} - \mathbf{K}, \\ \varepsilon_o \dfrac{\partial \mathbf{E}}{\partial t} = \nabla \times \mathbf{H} + e n_o \mathbf{v} - \mathbf{J}, \\ m n_o \dfrac{\partial \mathbf{v}}{\partial t} = -\nabla p - e n_o (\mathbf{E} + \mathbf{B} \times \mathbf{v}) + \mathbf{F}, \\ \dfrac{1}{n_o m v_o^2} \dfrac{\partial p}{\partial t} = -\nabla \cdot \mathbf{v} + \Phi \end{cases} \qquad (1)$$

in Q, where $\mathbf{K}$ is the magnetic current; $\mathbf{J}$ is the electric current; $\mathbf{F}$ is the external body force, Φ is the flux, $\mathbf{B}$ is the magnetic induction; n_o and v_o are the mean density of electrons and mean speed of electrons respectively. Parameters e, ε_o and μ_o represent the charge of the electron, the dielectric constant and the magnetic permeability, in vacuum, related by the condition

$$c^2 \stackrel{\text{def}}{=} \frac{1}{\varepsilon_o \mu_o} > v_o^2.$$

Here ∇ is the usual nabla differential operator

$$\nabla \stackrel{\text{def}}{=} \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3} \right).$$

The initial conditions are

$$\begin{cases} \mathbf{H}(x,0) = \mathbf{h}(x), \quad \mathbf{E}(x,0) = \mathbf{e}(x) \\ \mathbf{v}(x,0) = \boldsymbol{\nu}(x), \quad p(x,0) = P(x) \end{cases} \text{in } \Omega, \qquad (2)$$

where $\mathbf{h}(x)$, $\mathbf{e}(x)$, $\boldsymbol{\nu}(x)$ and the scalar function $P(x)$ are known in Ω, while on the boundary $\partial\Omega$, we have

$$\begin{cases} \mathbf{E} \times \mathbf{H} \cdot \mathbf{n} \le 0 \\ p\mathbf{v} \cdot \mathbf{n} \le 0 \end{cases} \text{in } \partial\Omega \times I, \qquad (3)$$

being $\mathbf{n}$ the outward normal vector.

For every cube $K \subset \mathsf{R}^3$, the rate of the energy in the domain $\Omega \cap K$ at time t, is

$$\frac{1}{2} \int\limits_{\Omega \cap K} \det \left(\mu_o \mathbf{H}^2 + \varepsilon_o \mathbf{E}^2 + n_o m \mathbf{v}^2 + \frac{p^2}{n_o m v_o^2} \right) dx.$$

If this integral converges to a finite value, for any K, then the plasma state is called with finite energy, or with locally finite energy if it converges only for a bounded measurable set K.

2. HYPERBOLIC SYSTEMS

Hyperbolic system of differential equations might be expressed as

$$E(x) \frac{\partial \mathbf{u}}{\partial t} = A^i(x) \frac{\partial i}{\partial \mathbf{u}} + B(x)\mathbf{u} + \mathbf{f}(x,t) \qquad (i = 1, \ldots, n), \qquad (4)$$

where the $m \times m$ matrices

$$E = (E_{\alpha\beta}), \quad A^i = (A^i_{\alpha\beta}), \quad B = (B_{\alpha\beta}) \qquad (\alpha, \beta = 1, \ldots, m),$$

and the m vectors

$$\mathbf{u} = (u_\alpha) \quad \mathbf{f} = (f_\alpha) \quad (\alpha = 1, \ldots, m)$$

are functions of $x = (x_i)$ and t. When E and A^i, $(i = 1, \ldots, n)$ are symmetric matrices and E is definitely positive, system (4) is an hyperbolic symmetric system.

If we define the electromagnetic state of a magnetohydrodynamics system (plasma) by the function $\mathcal{H} = [\mathbf{H}, \mathbf{E}, \mathbf{v}, p]$, and the field source by $\mathcal{F}(x,t) = [-\mathbf{K}, -\mathbf{J}, \mathbf{F}, \Phi]$, system (1) takes the form (see e. g. [9]) of an hyperbolic symmetric system like (4), assuming $n = 3$, $m = 10$ and with $\mathbf{u} = \mathcal{H}$, $f = \mathcal{F}$:

$$E(x) \frac{\partial \mathcal{H}}{\partial t} = A^i(x) \frac{\partial i}{\partial \mathcal{H}} + B(x)\mathcal{H} + \mathcal{F}(x,t) \text{ in } \Omega \qquad (i = 1, \ldots, n). \qquad (5)$$

The first order differential operator A acting on the class of C^1-functions is defined as

$$A\mathcal{H} \stackrel{\text{def}}{=} A^i \frac{\partial i}{\partial \mathcal{H}} \quad (i = 1, 2, 3)$$

and verifies the condition on the formal adjoint $A^\star = A$. The initial state (2) is represented by

$$\mathcal{H}(x,t)|_{t=0} = \mathcal{H}^o, \qquad \mathcal{H}^o \stackrel{\text{def}}{=} [\mathbf{h}(x), \mathbf{e}(x), \boldsymbol{\nu}(x), P(x)], \qquad (6)$$

and the boundary conditions (3) might be reduced to

$$\mathcal{S}(\mathcal{H}(x,t)) \le 0 \text{ in } \partial\Omega \times I, \qquad (7)$$

being $\mathcal{S}$ an algebraic operator.

3. SPACE OF SOLUTIONS

With respect to (4), let us consider only the functions $u : D \longrightarrow H$, defined in an arbitrary domain

$D\subset \mathsf{R}^n$, with values into an Hilbert separable space H, where the inner product is

$$\langle f,g\rangle_H \stackrel{\text{def}}{=} \int\limits_D f(x)g^\star(x)dx,$$

and the norm of a function is

$$||f||_H = \langle f, f^\star\rangle_H^{1/2}.$$

The space

$$L_2(D,H) = \Big\{ u : u \text{ is measurable in } H,$$

$$\int\limits_D ||u||_H^2 dx < \infty \Big\}$$

is the Hilbert space, moreover

$$L_2^{loc}(D,H) = \Big\{ u : u \in L_2(K\cap D, H)$$

$$\forall \text{ bounded measurable set } K\subset \mathsf{R}^n \Big\}$$

is the space of locally square-integrable functions. We define the linear subspaces:

$$L_2(\Lambda, D, H) \stackrel{\text{def}}{=} \Big\{ u : u \in L_2(D,H)$$

$$\text{and } \Lambda u \in L_2(D,H') \Big\},$$

$$L_2^{loc}(\Lambda, D, H) \stackrel{\text{def}}{=} \Big\{ u : u \in L_2^{loc}(D,H)$$

$$\text{and } \Lambda u \in L_2^{loc}(D,H') \Big\},$$

where Λ is a linear differential operator $\Lambda : H \longrightarrow H'$ with bounded and measurable coefficients, and, in particular, it is

$$H^1(I,H) \stackrel{\text{def}}{=} L_2(\det, I, H).$$

Let Γ_1, Γ_2, Γ_3, Γ_4 be closed linear subspaces of $L_2(\nabla\times,\Omega,\mathsf{R}^3)$, $L_2(\nabla\times,\Omega,\mathsf{R}^3)$, $L_2(\nabla\cdot,\Omega,\mathsf{R}^3)$, $L_2(\nabla,\Omega,\mathsf{R})$ respectively, so that $\mathbf{H}\in\Gamma_1$, $\mathbf{E}\in\Gamma_2$, $\mathbf{v}\in\Gamma_3$, $p\in\Gamma_4$, and

$$\Gamma \stackrel{\text{def}}{=} \Gamma_1\times\Gamma_2\times\Gamma_3\times\Gamma_4$$

be a closed linear subspace of $L_2(A,\Omega,\mathsf{R}^{10})$. The space

$$F = L_2(I,\Gamma)\cap H^1\left(I, L_2(A,\Omega,\mathsf{R}^{10})\right)$$

represents the class of functions $\mathcal{H}(x,t)$ for which $E\partial\mathcal{H}/\partial t$, $A\mathcal{H}$, $B\mathcal{H}$ exist in $L_2(Q;\mathsf{R}^{10})$, and satisfies the conditions (6) – (7), in the sense that $\mathcal{H}(t)\in\Gamma$ for almost all $t\in I$. In particular, we have the following solutions (for the weak solutions see [9])

Definition 1 *$\mathcal{H}$ is a FE-solution of system (5) with boundary conditions (6) – (7) in Γ and $\mathcal{F}(x,t)\in L_2(Q;\mathsf{R}^{10})$, $\mathcal{H}^o(x)\in L_2(\Omega)$ given, if $\mathcal{H}(t)\in F$ satisfies:*

$$\begin{cases} E\dfrac{\partial\mathcal{H}}{\partial t} = A\mathcal{H}+B\mathcal{H}+\mathcal{F} & \text{almost everywere in } Q,\\ \mathcal{H}(0)=\mathcal{H}^o & \text{almost everywere in } \Omega. \end{cases} \qquad (8)$$

Definition 2 *$\mathcal{H}$ is a LFE-solution of system (5) with boundary conditions (6) – (7) in Γ and $\mathcal{F}(x,t)\in L_2^{loc}(Q,\mathsf{R}^{10})$, $\mathcal{H}^o(x)\in L_2^{loc}(\Omega)$ given, if $\mathcal{H}(t)\in F^{loc}$ satisfies system (8),*
where

$$F^{loc} \stackrel{\text{def}}{=} \Big\{ \mathcal{H} : \mathcal{H}\in L_2\big(I, L_2(A, K\cap\Omega)\big)\cap$$

$$\cap H^1\big(I, L_2(A,K\cap\Omega)\big)$$

$$\forall \text{ bounded measurable set}$$

$$K\subset\mathsf{R}^3 \text{and } \mathcal{H}(t)\in\Gamma^{loc}\ \forall t\in I\Big\}.$$

In the following we will consider a numerical approximation of a FE-solution (8), in the Haar wavelet bases.

4. HAAR WAVELETS

In order to obtain an L_2-approximation for the solution of problem (8), we exploit the collocation method in a wavelet bases (see e. g. [10] and references therein), restricting (without any loss of generality) to dimension 1 ($\mathsf{R}^n \to \mathsf{R}$). We consider the Haar family of wavelets [1, 2, 11], based on two fundamental functions: the so-called scaled function $\Phi(x)$ and the (mother) wavelet $\Psi(x)$. From the latter, by the dilation (depending on a scale factor n) and the translation (depending on k), we will derive the wavelet bases

$$\Psi_{n,k}(x) \stackrel{\text{def}}{=} \left\{2^{\frac{n}{2}}\Psi(2^n x - k)\right\}_{k,n\in Z}, \qquad (9)$$

for the L_2-functions.

Let $D\subset\mathsf{R}$ and $\mathbf{1}_D(x)$ the rectangle function (with compact support)

$$\mathbf{1}_D(x) = \begin{cases} 1 & (x\in D),\\ 0 & (x\notin D),\end{cases}$$

the Haar scaling function is

$$\Phi(x) \stackrel{\text{def}}{=} \mathbf{1}_{[0,1)}(x),$$

and the Haar (mother) wavelet $\Psi(x)$ is

$$\Psi(x) = \mathbf{1}_{[0,\frac{1}{2})}(x) - \mathbf{1}_{[\frac{1}{2},1)}(x).$$

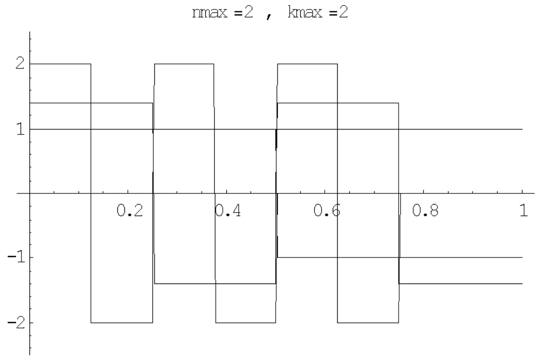

Fig. 1. Haar wavelet bases in W_3 ($D_{n,k} \subseteq [0,1]$, $n=2$, $0 \leq k \leq 2$)

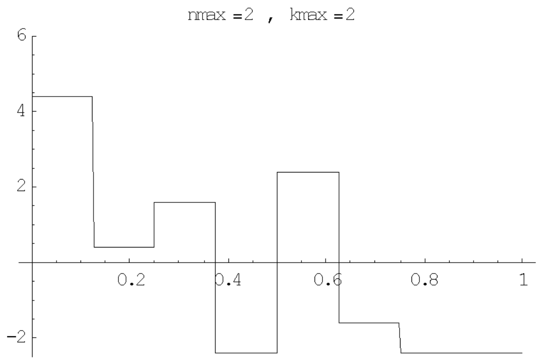

Fig. 2. Composition of the Haar wavelet bases in W_3 ($D_{n,k} \subseteq [0,1]$, $n=2$, $0 \leq k \leq 2$)

These functions are related by the recursive formula [1,2]:

$$\begin{cases} 2^{\frac{n}{2}}\Phi(2^n x - k) = \\ \quad = \Phi(2^{n+1}x - k) + \Phi(2^{n+1}x - (k+1)), \\ 2^{\frac{n}{2}}\Psi(2^n x - k) = \\ \quad = \Phi(2^{n+1}x - k) - \Phi(2^{n+1}x - (k+1)). \end{cases} \tag{10}$$

($n, k \in \mathcal{Z}$) From the mother wavelet $\Psi(x)$, by scaling and translating, it is possible to define the set of wavelets (9):

$$\Psi_{n,k}(x) \stackrel{\text{def}}{=} \begin{cases} 1, & x \in \left[\frac{k}{2^n}, \frac{k+1/2}{2^n}\right), \\ -1, & x \in \left[\frac{k+1/2}{2^n}, \frac{k+1}{2^n}\right), \\ 0, & \text{elsewhere}\ . \end{cases} \tag{11}$$

which are compact functions on the dyadic intervals

$$D_{n,k} \stackrel{\text{def}}{=} \left[\frac{k}{2^n}, \frac{(k+1)}{2^n}\right), \quad n, k \in Z$$

We will see in section 5 that for a fixed n the family of wavelets $\Psi_{n,k}$ are a bases for $L_2(D, \mathsf{R})$ functions, so that each point $(\overline{x}, \overline{y})$ of $\mathsf{R}\times\mathsf{R}$ might be expressed by $(\overline{x}, 2^{\frac{n}{2}}\Psi(2^n\overline{x} - k))$ for suitable (n, k). Thus each point of $\mathsf{R}\times\mathsf{R}$ becomes function of two parameters (n, k), in the functional space whose generator are $\{2^{\frac{n}{2}}\Psi(2^n x - k)\}$, which are an orthonormal basis for $L_2(D, \mathsf{R})$.

5. ADAPTIVE SPACES

Let $\{V_n\}_{n\in Z}$ be the subset of $L_2(D, \mathsf{R})$ defined as the set of functions $f(x)$ of compact support on $D_{n,k}$

$$V_n \stackrel{\text{def}}{=} \Big\{ f(x) \in L_2(D_{n,k}, \mathsf{R}) : f(x) = \text{ const } \forall x \in D_{n,k}, \\ f(x) = 0 \ \forall x \notin D_{n,k} \Big\}.$$

Functions $f(x)$ and subsets V_n fulfill the axioms of multiscale resolution analysis [1, 11, 12]. According to them, $\{\Phi(x-k)\}_{k\in Z}$ is an orthonormal basis in V_0 and, for arbitrary n, adding the complementary subspace (of wavelet) W_n,we have

$$V_{n+1} = V_n \oplus W_n,$$

where $\oplus$ is the direct sum of orthogonal subspaces. As a consequence the space $L_2(D, \mathsf{R})$ is "reconstructed" as direct sum of orthogonal subspaces W_n of wavelets

$$L_2 = \oplus_{n\in Z} W_n.$$

In each

$$W_n = \text{span}_{k\in Z, x\in \mathsf{R}} \Psi_{n,k}(x)$$

the basis functions are the functions $\Psi_{n,k}(x)$ (at fixed n) of (9); for variable n, the all set of functions $\Psi_{n,k}(x)$ represents an orthonormal basis for L_2 (fig. 1, 2). As a consequence, any L_2-function $f(x)$, owns a spectral decomposition in Haar wavelets (fig. 3)

$$f(x) = \sum_{n,k\in Z} \beta_k^n 2^{\frac{n}{2}}\Psi(2^n x - k), \tag{12}$$

where

$$\beta_k^n \stackrel{\text{def}}{=} \int_{-\infty}^{+\infty} f(x) 2^{\frac{n}{2}}\Psi(2^n x - k) dx. \tag{13}$$

When $\Psi(x)$ is given, the functional dependence on the factor $2^{\frac{n}{2}}$, through the parameter n, produces a scaling of $\Psi(x)$ while a non trivial k translates $\Psi(x)$ either rightward ($k>0$), or leftward ($k<0$). Furthermore $\Psi(x)$ is a well localized function in the sense that

$$|\Psi^{(n)}(x)| \leq C \exp(-\alpha|x|) \ \forall x \in \mathsf{R},$$

$$n \leq N \leq \infty, \quad \alpha > 0.$$

The bases $\Psi_{n,k}(x)$ extend to the whole real line, however if we have a function $f(x)$ defined in a finite domain Ω of R, we can introduce a function $\tilde{f}(x)$ that coincides with $f(x)$ in Ω and has a compact support in Ω, i.e. vanishes out of Ω.

We notice that (see [10]):

- for $\varepsilon>0$ there exists a value $M<\infty$ such that for $n>0$ and $k\in Z$, there exists a constant α such that $||\Psi_{n,k}(x)-\alpha||_{L_2(D,\mathsf{R})}<\varepsilon$;
- for $\varepsilon>0$, there exists a value n such that $||\Psi_{n,k}(x)||_{L_2(D,\mathsf{R})}<\varepsilon$.

In other words, in the numerical approximation of a function $f(x)$, built on wavelets, there exists a finest "level" n, fitting $f(x)$.

6. COLLOCATION WAVELET METHOD

Let $\Omega\subset\mathsf{R}$ be a finite domain discretized in M subintervals at points

$$x_j = \frac{j}{M} \quad (j=0,\ldots,M-1),$$

$u(x,t)$ a one-parameter (t) function, sampled at the set of locations $\{x_j\}$, and the corresponding sampled values

$$u_j^t \stackrel{\text{def}}{=} u(x_j,t).$$

Using a pointwise approximation, and the wavelet bases (9), we have

$$u(x,t) \cong \sum_{j=0}^{M} p_j(x,t) u_j^t, \tag{14}$$

where

$$p_j(x,t) \stackrel{\text{def}}{=} \begin{cases} \delta_{jk} = \left\langle \begin{array}{l} 0, \ k\neq j, \\ 1, \ k=j \end{array} \right. \\ \qquad \text{at} \ \ x=x_k, \\ \\ \alpha_0^0(t) + \sum\limits_{n=0}^{N} \sum\limits_{k=0}^{2^M-1} \beta_k^n(t) 2^{\frac{n}{2}} \Psi(2^n x - k) \\ \qquad \text{at} \ \ x\neq x_k. \end{cases} \tag{15}$$

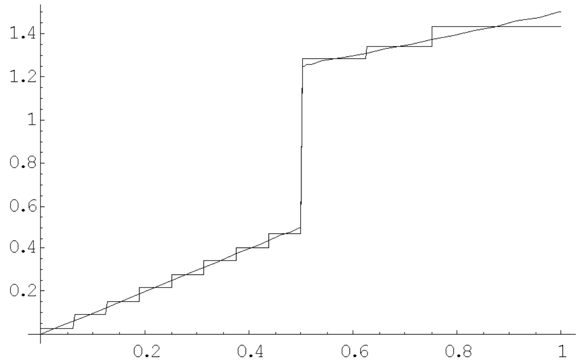

Fig. 3. Haar wavelets representation of a C^0-function ($n\leq 3$, $k\leq 3$)

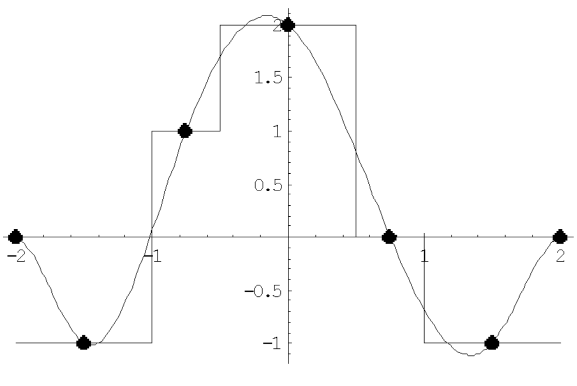

Fig. 4. Haar representation and the corresponding spline

For a fixed t the wavelet coefficients $\beta_k^n(t)$ are given by Eq. (13), while the first coefficient $\alpha_0^0(t)$ is given by an equation similar to (13) in which $\Phi(x)$ takes the place of $\Psi(2^n x-k)$. Thus the value of $u(x,t)$ in $x\neq x_j$ (at fixed time t) is determined by the values u_j^t (supposed to be known) at the nodal points x_j and by the values of the wavelet coefficients α_0^0, β_k^n. These coefficients, at fixed time t, might be (alternatively) easily computed by a FHT, Fast Haar Transform (see e.g. [1,5,7,10]).

Since we are using the Haar wavelets having constant values, the formal derivatives, with respect to x, of (14) (according to (15)) trivially vanish. Thus, in order to get a similar wavelet representation (14)–(15), for the x-derivatives of $u(x,t)$ we suggest to "transform" the piecewise function (14) in a non trivially differentiable function, using splines. To this end, one has just to pick the middle points [4] of the subintervals in Eq. (11), and, together with the boundary points x_0, x_M, to build up a spline (see

Table. 3-order spline corresponding to fig. 4

$y = 52.6263 \ + 96.7111x + 56.9637x^2 + 10.8824x^3$	$x \in [-2, -3/2]$
$y = 0.273795 \ - 7.99393x - 12.8397x^2 - 4.6295x^3$	$x \in [-3/2, -3/4]$
$y = 2. \ - 1.08911x - 3.63324x^2 - 0.537753x^3$	$x \in [-3/4, 0]$
$y = 2. \ - 1.08911x - 3.63324x^2 + 2.03977x^3$	$x \in [0, 3/4]$
$y = 2.07578 \ - 1.39221x - 3.2291x^2 + 1.86015x^3$	$x \in [3/4, 3/2]$
$y = 35.9926 \ - 69.2259x + 41.9934x^2 - 8.18928x^3$	$x \in [3/2, 2]$

fig. 4 and the table).

By deriving the spline we obtain a function that, according to Eq. (12) can be expressed into a series of Haar wavelets (an alternative numerical algorithm is reported in [10]). Thus also the derivative $\partial u/\partial x$ can be expressed by an equation having the same form of (14)

$$\frac{\partial u(x,t)}{\partial x} \cong \sum_{j=0}^{M} q_j(x,t) u_j^t, \qquad (16)$$

where the coefficients q_j have the same form of (15).

7. WAVELET INTERPOLATION

In the one-dimensional problem ($\Omega \subset \mathsf{R}$, $\partial\Omega = \{x_0, x_1\}$) with finite energy Eq. (4) becomes

$$E(x) \sum_{j=0}^{M} \left[\frac{d\alpha_0^0(t)}{dt} + \right.$$

$$\left. + \sum_{n=0}^{N} \sum_{k=0}^{2^M-1} d_M \frac{d\beta_k^n(t)}{dt} 2^{\frac{n}{2}} \Psi(2^n x - k) \right] u_j^t =$$

$$= \mathcal{A}(x) \sum_{j=0}^{M} \frac{dq_j(x,t)}{dt} u_j^t +$$

$$+ B(x) \sum_{j=0}^{M} p_j(x,t) u_j^t + \mathbf{f}(x,t)$$

$$(i = 1, \ldots, n),$$

having taken into account equations (14) – (16). The above after some trivial manipulations, based on the Galerkin method, gives rise to an ordinary first order differential system in the unknown functions

$$\boldsymbol{\beta} \stackrel{\text{def}}{=} \{\alpha_0^0(t), \beta_k^n(t)\},$$

alike

$$\frac{d\boldsymbol{\beta}}{dt} = F(\boldsymbol{\beta}, x, t) \qquad (17)$$

with initial conditions

$$\boldsymbol{\beta}(t)|_{t=0} = \boldsymbol{\beta}_0,$$

being $\boldsymbol{\beta}_0$ the wavelets coefficients of the initial function $u(x,0)$. A further numerical approach such as Eulero or Runge – Kutta might help to solve differential system (17).

CONCLUSIONS

The wavelet representation method applied to the solution of the fundamental equations of magnetohydrodynamics which are based on a hyperbolic system of differential equations is discussed. The explicit numerical method is sketched.

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